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BIDDING FOR CONTRACT GAMES
APPLYING GAME THEORY TO ANALYZE
FIRST PRICE SEALED BID AUCTIONS

by

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BIDDING FOR CONTRACT GAMES
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TO ANALYZE FIRST PRICE SEALED BID AUCTIONS

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This study analyzed the first price sealed bid auction (FPSBA) using computer simulations. The first price sealed bid auction is a static Bayesian game with incomplete information. These games have a well-defined symmetric Bayesian Nash equilibrium. The existence of the equilibrium makes it possible to find the bidders' equilibrium strategies. The equilibrium strategy maximizes the bidders' profit. This thesis assumes, (1) the bidders act rationally and have private information about their production cost, (2) the bidders' preferences and information are symmetric, (3) the buyer committed not to deviate from the auction rules, even if a deviation would be profitable. Considering these assumptions and the equilibrium strategy, this Thesis constructed a FPSBA model. The model was transformed into an algorithm and coded in Visual Basic language. The code was used to simulate the FPSBA in different scenarios. The simulation showed the bidders' behavior and identified factors affecting the bidders' decision during bid preparation. Critical factors include the cost distribution and number of bidders. The concluding chapter presents the analytical results.

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I. INTRODUCTION

Hungary assumes that establishing a stable security environment on the European Continent is in her vital interest. The transformation to a stable democracy in Central and Eastern Europe has contributed to the security of the region and Europe as a whole. At the same time, the social tensions within the transforming countries, ethnic and religious diversities rooted in history, require the permanent development of a new security architecture.

Hungary's strategic goal is to gain full membership in the existing international security and defense organizations, including the Western European Union and NATO. The country's geostrategic position, fundamentals, material and human resources justify maintaining armed forces with self-defense capabilities comparable to similar Central and Eastern European countries. However, Hungary deems her armed forces to be the very last means of self-defense.

The Republic of Hungary has made a commitment to join the North Atlantic Treaty Organization (NATO) in the near future, and to transform her armed forces into a modern force capable of meeting the NATO's military standards.

> In due course we are preparing ourselves for participating in missions deriving from NATO basic principles, such as the common defense pronounced in Article 5 of the Washington Treaty, furthermore to contribute to peacekeeping operations, humanitarian and other new missions. Regarding the modernization of the Hungarian Defense Forces (HDF), we intend to harmonize ourselves with the major polices and priorities indicated in the NATO Enlargement Study. [Ref. 1]

Modernization of the Hungarian Defense Forces (HDF) has already started. The winner of the low-level air-defense system tender was announced in February 1997. A new request for proposal will be solicited for modernizing the air surveillance systems in the near future. Hungary is considering rejuvenating her Air Force and other outdated military hardware as well. These initiatives require new practices of procurement source selection that use the country's scarce resources more efficiently.

The Public Procurement Law of Hungary, enacted in 1995, defines the requirement for public acquisition of goods and services in Hungary. The Public

Procurement Law obligates the public fund users to select suppliers on a competitive bases. The Law outlines the methods of public procurement and it states that "all contracts exceeding value of HUF 5,000,000 [Apr. $35,000] should be awarded to the winner of a publicly announced tender." [Ref. 2] The winner is the best bid when all other conditions are equal.

Market conditions in Hungary are changing day after day. New potential sources for military procurement come into sight and disappear. The international market, where the HDF expects to acquire its military equipment, differs geographically and economically as well. It includes all the major defense industry players in NATO and non-NATO countries. As the economic conditions are diverse, so various are the offers coming from the bidders. A fair and efficient method of bid selection can increase efficiency in using the HDF scarce resources.

Auctions are one of the market institutions that provide a competitive environment for public procurement. Auctioning practice and theory classify auctions in two major groups; first and second price auctions. In a first-price auction, the bidders submit their bids simultaneously and the lowest bidder wins the auction at the bid price. Rules of second-price sealed bid auctions stipulate that the bidder submitting the lowest (highest) bid win the auction and pay the second lowest (highest) price.[Ref. 3] The first-price sealed bid auction is one of the basic tools for source selection in the Government procurement and in the HDF in particular. While the second-price sealed bid auction has useful theoretical properties, it is rarely used in practice and it has never been used in Hungary.

To understand the strategic behavior of the bidders and to find opportunities to influence the recent auctioning practice in the HDF, this thesis proposes a model of the FPSBA. The model describes the strategic behavior of the bidders and their actions in conditions similar to the actual bidding conditions, based on findings of Game Theory.

A. AN OVERVIEW OF THE HDF PROCUREMENT ACTIVITY

The HDF has a composite procurement system. It consists of two major subsystems; they are the unit-level, decentralized procurement subsystem and the centralized acquisition subsystem. The decentralized subsystem buys goods and services from the local market (food for the soldiers, building maintenance service for barracks buildings and similar activities). The volume of local procurement is significant and it is approximately 15 - 20 % of HDF's over all maintenance costs. The centralized procurement system acquires goods and services for the HDF, to keep up the combat readiness of the troops and to provide planed development of military hardware.

To comply with the Public Procurement Law of Hungary, the military procurement system uses competitive acquisition from abroad and from the domestic market. As the market economy advances in Hungary, more domestic potential suppliers come into view for the HDF. However, in some areas of military procurement, sole source acquisition remains the practice. The complexity of market conditions and the diverse assortment of the goods and services under procurement require proper acquisition practices from the HDF. Understanding the behavioral patterns of the potential supplier, their motivations and strategic behavior better prepares the buyer. The more prepared the buyers, the better are their chance of concluding effective and efficient business deals.

The HDF and the acquisition organization of the Hungary's Ministry of Defense use a wide variety of competitive procurement policies and practices. Sealed bid auctions have been extensive applied recently by the HDF. They have been used in purchasing major defense equipment, low unit cost but high volume materials, and various services as well. The Hungarian acquisition system generally uses first price sealed bid auctions. In some areas of procurement, however, open bidding practices are also used. The developing acquisition policy is considering new forms and activities.

B. RESEARCH QUESTIONS AND RESEARCH METHODS

The primary research question for this thesis is as follows:

<u>How do profit maximizing suppliers choose their bids in a competitive environment?</u>

This thesis research examines the following secondary research questions:

1. Does the FPSBA have a game equilibrium, and if it has, what are the equilibrium strategies of the bidders?
2. Do the bidders have a dominant strategy in First Price Sealed Bid Auctions?
3. How does the change in bidders' numbers affect the outcome of the FPSBA?
4. How does the cost distribution affect the outcome of the FPSBA?
5. How can HDF use the findings of this Thesis in their acquisition practice?

C. SCOPE OF THE RESEARCH AND ORGANIZATION OF STUDY

This thesis is limited to applying the FPSBA to those areas where the HDF has a limited number of responsive and responsible potential suppliers. The limitation also applies to the goods and services the HDF intends to acquire. The goods and services should be accurately specified and the HDF or the Acquisition Office must not deviate from these specifications.

On the bases of conducted research, this thesis:

1. Will review the contracting practice of the HDF to find those areas where the bidding for contract simulations can take place,
2. Will review the theory of auctioning activity and the game theory providing the background for constructing a model of the FPSBA,
3. Will develop a computer simulation to analyze the FPSBA and experiment with the model.

4. Will provide recommendations to use computer simulations of FPSBA auctions in the HDF's contracting practice.

The thesis has been organized in four chapters as follows. Chapter II surveys the development of auctioning theory. Considering competitive selling auctions, the chapter defines the bidding for contract game in normal-form, and the existence of a Bayesian Nash equilibrium in these games. The Chapter develops bidding functions for two and three bidders in the bidding for contract game. The bidding function is developed when the bidders' production costs are distributed uniformly, and for the case where the cost distribution follows a triangular distribution.

Chapter III describes the experiments conducted with the constructed model. Computer simulation has been used as the basic experimentation tool. The chapter gives a summary of the settings and methodology in which the experiments were organized; the chapter also analyzes the results.

Conclusions and recommendations are provided in Chapter IV. This chapter addresses each of the primary and secondary research questions posed in this chapter, and provides concluding remarks about the feasibility of HDF using computer simulations in acquisition practice. Additionally, it suggests areas for further research.

D. DEFINITIONS

Auction -- as used throughout this thesis, refers to "a market institution with an explicit set of rules determining resource allocation and price on the basis of bidding from the market participants."[Ref. 4]

Strategy -- as used throughout this thesis, refers to the definition, given by von Neuman and Morgenstern, founders of the Game Theory "a complete plan: a plan that specifies what choices [the player] will make in every possible situation."[Ref. 5]

Bidder -- as the name implies, refers to a responsive and responsible prospective supplier, where:

1. A responsive supplier implies the offeror has the ability to comply with the specifications, quantities to be delivered, and terms and conditions encountered in the contract,

2. responsive supplier:

a) has adequate financial resources to perform the contract or the ability to obtain such resources

b) is able to comply with the contracted delivery schedule,

c) has the necessary organization, experience, and technical skills.

d) has the necessary production, construction or technical equipment and facility to perform the contract obligations, [Ref. 5]

Buyer -- as the name implies, refers to a government agency (the HDF in particular) soliciting a request for proposal to submit a bid for a specified contract.

Bidding for contract game -- as used throughout this thesis, refers to the definition of a single stage static game with incomplete information played by the bidders. The buyer is not actively involved in the game.

II. BACKGROUND

This Chapter will explore the basis of auctioning theory. The first part of the Chapter will analyze the games to which auctioning activity applies. The bounding of the game to a class limits the scope of the research. The second part of the Chapter will analyze bidding for contract games. The third part of this Chapter will summarize the basics of the order statistics, which will later be used to control the simulation outcome. After identifying the main characteristics of the static non-cooperative games of imperfect information, this Chapter will conclude with the normal-form representation and the equilibrium strategies of the bidding for contract games.

Organized auctioning from which sealed bidding originates dates back to the ancient times. One of the earliest reports of an auction was given by Herodotus. He described the sale of women to be wives in Babylon around the fifth century BC. In China, as early as the seventh century AD, the personal belongings of deceased Buddhist monks were sold at auction.[1]

In modern times, auctioning practice distinguishes two major families of auctions. The first is selling auctions held to market goods and to sell them at the highest possible price. In other class of auctions, the buyer purchases goods or services from competing suppliers, represented by bidders. This thesis will call the later group 'bidding for contract" auctions. Using this terminology, we can distinguish the two types of actions. Game theory links these two auction types to the same class of games, called static non-cooperative games with imperfect and incomplete information.

A. THEORETICAL PERSPECTIVES OF THE FPSBA

William Vickrey laid the foundation for auction theory in his remarkable 1961 paper.[Ref. 3] Paul Milgrom and Robert Weber provided a general framework to analyze the competitive bidding process.[Ref. 8] McAfee and MacMillan gave a comprehensive

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[1] These and other historical references can be found in Cassady [Ref. 7]

survey of auction theory in 1987.[Ref. 4] Several other authors have explored auctioning, mostly concentrating on the selling auctions. Charles Holt [Ref. 9] and William Samuelson [Ref. 10] developed the theory of competitive bidding for contracts. The literature describes four basic types of auction: the English auction (also called oral open or ascending auction); the Dutch auction (also called oral open descending auction); the first-price sealed-bid auction; and the second-price sealed-bid (or Vickrey) auction.

The English auction is the most frequently used form for selling goods. The English auction prescribes that the price of the goods up for auctions is successively raised until only one bidder remains. The rising price can be announced by an auctioneer or the bidders themselves, or by having the bids submitted electronically with the current highest price posted.

The Dutch auction is the reverse of the English auction. The auctioneer calls an initial price, high enough to be above the bid of any of the participants. Then the price is lowered until one bidder accepts the current price. From game theory point of view, the distinguishing feature of the English and Dutch auctions is that, the bidders always know the actual level of the highest bid. Both auctions are oral auctions. Although, these auction forms have their merits, they are not usually used in government contracting.

In a first price sealed bid auction, the potential sellers (buyers) submit sealed bids simultaneously, and the lowest (highest) bidder is awarded the contract for the price bid. With the sealed bid auction, each bidder has only one opportunity to bid. The difference between the first-price sealed bid auction and the English auction is that the bidders at the sealed bid auction cannot observe the rival's bids; they cannot revise and alter their bids correspondingly.

In the second price sealed bid auction, the bidder submitting the lowest (highest) bid wins the auction, but the winner will get (will pay) the second lowest (highest) price. [Ref. 4] First-price sealed bid auctions are one of the basic tools for source selection in Government procurement, and in the HDF. The second-price sealed bid auction has useful theoretical properties but is rarely used in practice.

Combinations of these basic auctioning forms have been used quite often, and additional rules are often imposed along with the basic auction rules. For example, a reservation price can be imposed, discarding all the bids if they are lower (higher) then the reservation price. The auctioneer may charge bidders an entry fee for the right to participate in the auction or the auctioneer may impose certain preferences on some of the bidders. An example of the late provision is a 10% price advantages for domestic bidders over the foreign bidders as stipulated by the Public Procurement Law of Hungry. [Ref. 2]

B. THEORY OF SEALED BID AUCTION GAMES

This section will define the representation of the bidding for contract game, after studying non-cooperative games with incomplete information. Defining the equilibrium in non-cooperative games with incomplete information will provide the theoretical basis to determine the bidders' equilibrium strategies. The appearance of the equilibrium in the game will facilitate surveying the bidders' strategies in FPSBA.

1. Normal-Form Representation Of Non-Cooperative Games with Incomplete Information

This thesis surveys non-cooperative games of incomplete information based on the study of R. Gibson [Ref. 12]. To develop a normal form representation of the static game with incomplete information, also called Bayesian games, we have to consider non-cooperative games of complete information. We can represent the normal form of an n player game with complete information, as follows:

$$G = \{ S_1,\ldots, S_n ; u_1,\ldots,u_n\}$$

where: S_i -- player i's strategic space
u_i -- player i's payoff function
when the player selects strategy $(s_1, s_2,\ldots, s_n)$

Further assuming that the players move simultaneously, than the non-cooperative game of complete information for a player simply involves choosing an action from the rationally available action space, A.

So, we can rewrite the normal-form of the game of complete information:

$$G = \{ A_1, \ldots, A_n, u_1, \ldots, u_n \}$$

where: A_i -- player i's action space
u_i -- player i's payoff function

In the static non-cooperative game of complete information, the timing of moves is as follows: the players simultaneously choose an action from the feasible set of actions A_i (i.e. player i chooses action a_i), and the payoff of $u_i (a_1, a_2, \ldots, a_n)$ is received.

The first step in developing the normal-form representation of the non-cooperative game with incomplete information is denoting the idea that each player has private information about his or her payoff. The players, however, are uncertain about other players' payoff functions. Let player i's possible payoff function be represented as $u_i (a_1, a_2, \ldots, a_n, t_i)$, where t_i is called the player i's type. t_i belongs to a set of possible types (type space). Each t_i type corresponds to a different payoff function. J. Harsányi [Ref. 12] first applied this notion for representing the payoff functions in games of incomplete information.

Given this definition of players' types, if players know their payoff functions it is equivalent to players knowing their type. Likewise, saying that player i may be uncertain about the other players' payoff function is equivalent to stating that player i is uncertain about the other players' type, denoted $t_{-i}(t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n)$. T_{-i} represents the set of all possible values of t_{-i}.

Players have beliefs about the other players' types. We denote the probability distribution of player i's belief about the probability distribution of other players' type, given that player i is type t_i, as $p_i(t_{-i} | t_i)$. In our analysis the players' types (cost) are identical and independent. In this case, $p_i(t_{-i} | t_i)$ does not depend on t_i, thus we can write

player i's belief as $p_i(t_{-i})$. There are contexts in which the other players' types are correlated; for simplicity we assume independence of the types.

We can derive the normal form representation of the non-cooperative game with incomplete information by joining the normal form representation of the non-cooperative game with complete information and the concepts of type and distribution of beliefs.

Definition [Ref. 12:pg. 148] the normal-form representation of an n-player non-cooperative game with incomplete information specifies the players' action space $A_1, \ldots, A_n$, their type space $T_1, \ldots, T_n$, their belief's $p_1, \ldots, p_n$, and their payoff functions $u_1, \ldots, u_n$. Player i's type, privately known by player i, determines player i's payoff function $u_i(a_1, a_2, \ldots a_n, t_i)$, and is a member of the set of possible types T_{-i}. Player i's belief $p_i(t_{-i} | t_i)$ describes the uncertainty about the n-1 other players' possible types t_{-i} given i's type t_i. We denote this game:

$$G = \{A_1,\ldots,A_n ; T_1,\ldots,T_n ; p_1, \ldots p_n ; u_1, \ldots ,u_n\}$$

Following Harsányi [Ref. 11], assume that the timing of the static Bayesian game is as follows:

1. nature draws a type vector $t_i = (t_1, \ldots, t_n)$, where t_i is drawn from the set of possible types T_i;
2. nature reveals t_i to player i but not to any other player;
3. the players simultaneously choose actions; and player i chooses action a_i from the feasible set A_i;
4. payoff $u_i(a_1, a_2, \ldots a_n, t_i)$ is received.

Introducing the fictional move by nature, in steps 1 and 2, produces a game with incomplete information that also satisfies the requirement for the games with imperfect information. Because nature only reveals player i's type to player i, but not to other players, the other players do not know the complete history of the game when taking their actions. This is a condition of the game with imperfect information.

Two technical assumptions complete our discussion about the normal-form representation of an n-player non-cooperative game with incomplete information. First, player i has private information about his type and also about the type of some other player(s). We cannot exclude this condition from the bidding for contract game explicitly. However, we can assume that the signal received about the other bidders' types is false, and that bidders do not consider this information in selecting their action.

The second technical point involves beliefs about the other players, $p_i(t_{-i} | t_i)$. It is assumed the timing of the game is common knowledge, so is the prior distribution p(t) from which nature draws the type vector $t = (t_1, \ldots, t_n)$. When nature reveals t_i to player i, i can compute the belief $p_i(t_{-i} | t_i)$ using Bayes' rule of conditional probability: [2]

$$p_i(t_{-i} | t_i) = p(t_{-i}, t_i) / p(t_{-i}) = p(t_{-i}, t_i) / \sum p(t_{-i}, t_i)$$

Furthermore, a player can compute the beliefs that the other players might hold. We assume that the type distribution is common knowledge in the bidding for contract game and takes the form of either a uniform or triangular probability distribution.

2. Definition of Bayesian Nash Equilibrium

To define the equilibrium in the static Bayesian game, we have to first define the strategic space of players. The players' strategy is a complete plan of action, specifying a worthwhile action in every circumstance in which the player might be engaged. In a static game with incomplete information, nature begins the game by drawing the players' type. Thus, a strategy for player i must specify a feasible action for each of players i's possible type.

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[2] Bayes' rule provides a formula for the conditional probability P(A| B) that event A will occur given that event B has occurred [Ref. 13]

Definition [Ref. 12: pg. 150] In the static Bayesian game $G = \{ A_1, \dots , A_n ; T_1, \dots , T_n ; p_1, \dots , p_n ; u_1, \dots , u_n\}$ a **strategy** for player i is a function $s_i(t_i)$, where for each type t_i in T_i, $s_i(t_i)$ specifies the action from the feasible set A_i that i would choose if type t_i is drawn by nature.

In static Bayesian games, unlike games with complete information, the strategic space is not given in the normal-form representation of the game. In the games of incomplete information, the strategic space is constructed from the type and action space. Player i's set of possible strategies is the set of all possible functions with range A_i and domain T_i.

It is seemingly unnecessary for the player i to specify actions for each of players i's possible type. Once nature has revealed a specific type to the player, that player should not be concern about the other possible types. However, in choosing a strategy, player i has to consider what the other player will do. What the other players will do largely depends on what they think player i will do if nature draws type t_i from T_i. Therefore, player i should consider what to do if each of the other players' types is drawn from the type space T_i once a specific type is revealed to player i.

The central idea of the Bayesian Nash game equilibrium is that each player's strategy must simultaneously be a best response to other players' strategy. This means that no player wants to change his or her strategy, even if the change involves only one action by one type.

Definition [Ref. 12:pg. 151] In the static Bayesian game $G = \{A_1, \dots , A_n ; T_1, \dots , T_n ; p_1, \dots , p_n ; u_1, \dots , u_n\}$ the strategies $s^* = (s_1^*, \dots s_n^*)$ are a Bayesian Nash equilibrium for each player i and for each type t_i in Ti, if $s_i(t_i)$ solves

$$\max \sum u_i(s_1^*(t_1), \dots s_{i-1}^*(t_{i-1}), a_i , s_{i+1}^*(t_{i+1}), \dots , s_n^*(t_n) ; t)p_i(t_{-i} \mid t_i)$$

It is straight forward to say that in the static Bayesian game with n players, if both the possible action space $(A_1, \dots , A_n)$ and type space $(T_1, \dots , T_n)$ are finite, there exists a Bayesian Nash equilibrium.

3. Normal-form Representation of Bidding for Contract Game

The FPSBA, from game theoretic point of view is a non-cooperative game with incomplete information. These games are often called Bayesian games. In these games, at least one of the players' payoff function is uncertain. In the bidding for contract game, the player type is the expected cost; different expected costs for the bidder will provide different payoffs. In FPSBA, each bidder knows their cost (valuation of the object on sale) but does not know any other bidders' cost (valuation). Bids are submitted in sealed envelopes, so we can assume that the bidders act simultaneously.

The normal-form representation of the bidding for contract game with two bidders competing for the contract is represented as:

$$u_i(b_i, b_j, c_i, c_j) = \begin{cases} b_i - c_i, & \text{if } b_i < b_j \\ 1/2(b_i - c_i) & \text{if } b_i = b_j \\ 0 & \text{f } b_i > b_j \end{cases}$$

When more than two players are competing for a contract the normal-form representation of the bidding for contract game is represented as:

$$u_i(b_i, \ldots, b_n, c_i, \ldots, c_n) = \begin{cases} b_i - c_i, & \text{if } b_i = \min(b1, \ldots, b_n) \\ 1/p(b_i - c_i) & \text{if } b_i = b_j \\ 0 & \text{if } b_i > \min(b1, \ldots, b_n) \end{cases}$$
$$i = 1, 2, \ldots, n - 1$$

Assuming no more than p bidders submit the same bid.

C. ORDER STATISTICS: AN OVERVIEW

This Section will survey the relevant order statistics theory using the guidelines provided in R. Hogg and A. Craig classic book of mathematical statistics [Ref. 13]. Theory of order statistics deals with the ranked values of a sample of random variables having drawn from a probability distribution. The ranking of the order statistics goes from the smallest to largest. Order statistics have some remarkable characteristic.

For example: properties of the order statistics do not depend upon the distribution from which the random sample has been drawn.

This thesis uses order statistics to analyze the outcome of the auctioning simulations. Bids and the bidders' production costs can be considered to be random variables drawn from a probability distribution. Ranking the submitted bids and the expected costs in ascending order we generates typical order statistics.

Let $X_1, X_2, X_3, \ldots, X_n$ denote a random sample from a continuos distribution, having probability density function of f(x) that is positive over the interval of $a < x < b$. Let Y_1 be the smallest of these X_i , Y_2 be the second smallest, ... , and Y_n be the largest. That is $Y_1 < Y_2 < Y_3, \ldots, Y_n$ represents $X_1, X_2, X_3, \ldots, X_n$ when they are arranged in ascending order. It can be proven that the joint probability distribution of $Y_1 < Y_2 < Y_3, \ldots, Y_n$ is given by

$$
\begin{aligned}
g(y_1, y_2, y_3, \ldots, y_n) &= n!*[f(x_1)*f(x_2)*f(x_3)*,\ldots, f(x_n)] \\
&\quad \text{if } a < y_1 < y_2 < y_3, \ldots, y_n < b \\
&= 0 \text{ elsewhere}
\end{aligned}
$$

The proof of this theorem is found in R. Hogg and A. Craig. [Ref. 13] The marginal probability density function represents the probability density function of one of the order statistics. This is given by

$$
\begin{aligned}
g_k(y_k) &= n!/(n-k)! *[F(y_k)]^{n-1}*[1-F(y_k)]^{n-k} * f(y_k) \text{ for } a < y_k < b \\
&= 0 \text{ elsewhere}
\end{aligned}
$$

The joint probability distribution function of the 1st and 2nd order statistics we can be used to calculate the expected differences between the sample members. Chapter III uses the computed results to control the simulation's outcomes. Examples of the calculations are given in Appendix B.

D. BIDDING FUNCTIONS UNDER THE UNIFORM COST DISTRIBUTION

A continuos random variable X has a uniform distribution if its probability density function is given by:

$$f(x) = \begin{cases} 1/(h - l) & l < x < h \\ 0 & \text{elsewhere} \end{cases}$$

The cumulative distribution function of the uniform distribution is given by:

$$F(x) = \begin{cases} 0 & x < l \\ (x - l)/(h - l) & l < x < h \\ 1 & x > h \end{cases}$$

Where: $l < h$ and
l - the lower limit of the distribution
h - the upper limit of the distribution

1. Bidding for Contract Games with Two Bidders

Assume that two bidders are competing for a contract and the bidders' production cost c_i has a uniform distribution over the range [0, 1]. Suppose Player j adopts the strategy $b(\cdot)$ and assume that $b(\cdot)$ is strictly increasing and differentiable. For a given value of c_i, player i's optimal bidding strategy solves

$$\max\ (bi - ci)^* \text{Prob}\{ bi < b(cj)\}$$

Let $b^{-1}(b_j)$ denote the cost the bidder must have to bid b_j. That is $b^{-1}(b_j) = c_j$ if $b_j = b(c_j)$. Since c_j is uniformly distributed on [0, 1], $\text{Prob}\{ b_i < b(c_j)\} = \text{Prob}\ \{b^{-1}(b_i) < c_j\} = 1 - b^{-1}(b_i)$. The first order condition for player i's optimization problem is therefor:

$$d[(b_i - c_i)^* (1 - b^{-1}(b_i))] / dbi = 0$$

Computing the derivative yields:

$$(1 - b^{-1}(b_i)) + (b_i - c_i)^*[d\ (1 - b^{-1}(b_i)) / dbi] = 0$$

The first order condition is an implicit equation for bidder i's best response to the strategy $b(\cdot)$ played by bidder j, given that the i's bidder cost is c_i. If the strategy $b(\cdot)$ is to

be a symmetric Nash equilibrium, we require that the solution of the first order condition be $b(c_i)$. That is, for each of bidder i's costs, the bidder does not want to deviate from the strategy $b(\cdot)$, given that bidder j plays this strategy.

To impose this requirement, we substitute $b_i = b(c_i)$ into the first order condition, yielding:

$$[1 - b^{-1}(b(ci))] + (b(ci) - c_i)*d\,[(1 - b^{-1}(b(ci))]/dbi = 0$$

where: $b^{-1}(b(ci)) = c_i$ and $d[1 - b^{-1}(b(ci))]/db_i = -1/b'(c_i)$

Thus, $b(\cdot)$ must satisfy the first order differential equation

$$1 - c_i - (b(ci) - c_i)*\ 1/b'(c_i) = 0$$

$$1 - c_i = (b(ci) - c_i)*\ 1/b'(c_i)$$

$$b'(c_i)*(1 - c_i) - b(ci) = -c_i$$

The left hand side of the equation can be rewritten as:

$$b'(c_i)*(1 - c_i) - b(ci) = d[(b(c_i)*(1 - c_i)]/dc_i$$

Integrating both sides of the equation yields:

$$\int d[(b(c_i)*(1 - c_i)]/dc_i = -\int c_i\, dc_i$$

$$b(c_i)*(1 - c_i) = -c_i^2/2 + k$$

To define k , we have to use the boundary conditions. These are $b(c_i) >= c_i$ particularly, if $c_i = 1$, $b(1)$ must be finite, and it is. Thus $k = c_i^2/2$ and $k = 1/2$.

Substituting the value of k into the equation after integration we find the bidding function for i.

$$b(c_i)*(1 - c_i) = (1 - c_i^2)/2 \qquad\qquad b(c_i) = (1 + c_i)/2$$

In the same way, we can define the reaction function for j. It will take the form:

$$b(c_j) = (1 + c_j)/2$$

Under the assumption that the players' strategies are strictly increasing and differentiable, we have a linear and symmetric Nash equilibrium in the two players bidding game.[3]

2. Bidding For Contract Games With Three Bidders

Assume that three bidders are competing for a contract and the bidders' production cost c_i has a uniform distribution over the range of [0, 1]. Suppose Player j and k adopt the strategy b(·) and assume that b(·) is strictly increasing and differentiable. For a given value of c_i, i's optimal bidding strategy solves:

$$\max(b_i - c_i)^* \text{ Prob}\{b_i < b(c_j) , b_i < b(c_k)\}$$

Let $b^{-1}(b_j)$ denote the cost the bidder must have in order to bid b_j and $b^{-1}(b_k)$ denote the cost bidder must have to bid b_k. That is $b^{-1}(b_j) = c_j$ if $b_j = b(c_j)$ and $b^{-1}(b_k) = c_k$ if $b_k = b(c_k)$.

Since c_j and c_k is uniformly distributed on [0, 1], $\text{Prob}(b_i < b(c_j)) =$ $= \text{Prob}(b^{-1}(b_i) < c_j) = 1 - b^{-1}(b_i)$ and $\text{Prob}(b_i < b(c_k)) = \text{Prob}(b^{-1}(b_i) < c_k) = 1 - b^{-1}(b_i)$. Therefor $\text{Prob}(b_i < b(c_j) , b_i < b(c_k)) = (1 - b^{-1}(b_i))^* (1 - b^{-1}(b_i)) = [1 - b^{-1}(b_i)]^2$.

The first order condition for player i's optimization problem is given by:

$$d[(b_i - c_i)^* (1 - b^{-1}(b_i))^2]/dbi = 0$$

Computing the derivative yields:

$$((1 - b^{-1}(b_i))^2 + (b_i - c_i)*2*((1 - b^{-1}(b_i))*d[(1 - b^{-1}(b_i)]/dbi = 0$$

Implying the same assumption as in case of two bidders, we substitute $b_i = b(c_i)$ into the first order condition, yielding:

$$[1 - b^{-1}(b(ci))]^2 + (b(ci) - c_i)^* 2^*[1 - b^{-1}(b(ci))]^*d[1 - b^{-1}(b(ci))]/dbi = 0$$

where: $b^{-1}(b(ci)) = c_i$ and $d[1 - b^{-1}(b(ci))]/db_i = - 1/b'(c_i)$

---

[3] A Nash equilibrium is called symmetric if the players' strategy are identical. That is, in a symmetric Nash equilibrium, there is a singe function b (c_i) such that bidder 1's strategy $b_1(c_1)$ is $b(c_1)$ and bidder 2's strategy $b_2(c_2)$ is $b(c_2)$, this singe strategy is the best response to itself. Of course, since bidders' costs will be different, their bid will be different even if both use the same strategy. [Ref. 12]

Thus, the b(·) must satisfy the following first order differential equation:

$$(1 - c_i)^2 - (b(ci) - c_i)*2* (1 - c_i)*1/b'(c_i) = 0$$

We can express this equation more conveniently as:

$$(1 - c_i)^2 = (b(ci) - c_i)*2* (1 - c_i)*1/ b'(c_i)$$
$$b'(c_i)*(1 - c_i)^2 - 2 b(ci) = - 2c_i$$

The left hand side of this equation can be rewritten as:

$$b'(c_i)*(1 - c_i)^2 - 2 b(ci) = 1/(1 - c_i)* d[(b(c_i)*(1 - c_i)^2]/dc_i$$

Integrating both sides of the equation, the right hand side by part

$$d[(b(c_i)*(1 - c_i)^2]/dc_i = c_i*[- 2 *(1 - c_i)]$$

$$\int d [(b(c_i)*(1 - c_i)^2]/dc_i = \int c_i*[- 2 *(1 - c_i)]dc_i$$

$$b(c_i)*(1 - c_i)^2 = c_i*(1 - c_i)^2 - \int(1 - c_i)^2 dc_i$$
$$b(c_i)*(1 - c_i)^2 = c_i*(1 - c_i)^2 + [(1 - c_i)^3]/3 + k$$

To eliminate k, we have to use the boundary conditions. That is, $b(c_i) >= c_i$; and particularly if $c_i = 1$, b(1) must be finite , and it is. So, k = 0

Substituting value of k into the above equation, we find the bidding function for i.

$$b(c_i) = c_i + (1 - c_i)/3 = [1 + 2*c_i]/3$$
$$b(c_j) = (1 + 2c_j)/3 \qquad \text{and} \qquad b(c_k) = (1 + 2c_k)/3 \text{ respectively.}$$

Under the assumption that the players' strategies are strictly increasing and differentiable, we derived a linear and symmetric Nash equilibrium in the three person bidding game.

E. BIDDING FUNCTIONS UNDER THE TRIANGULAR COST DISTRIBUTION

In general, a random variable X has a triangular distribution if its probability density function f(x) is given by

$$f(x) = \begin{cases} 2*(x - l)/(h - l)*(m - l) & l < x < m \\ 2*(n - x)/(h - l)*(h - m) & m < x < h \\ 0 & \text{elsewhere} \end{cases}$$

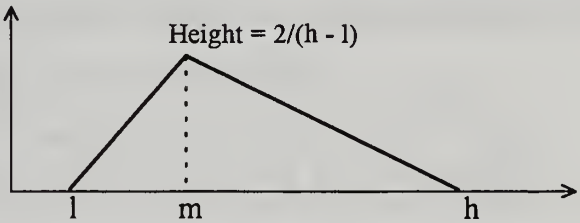

Figure 1 PDF of the triangular distribution

The cumulative distribution function F(x) of the triangular distribution is given by

$$F(x) = \begin{cases} 0 & x < l \\ (x - l)^2/(h - l)*(m - l) & l < x < m \\ 1 - (h - x)^2/ (h - l)*(h - m) & m < x < h \\ 1 & x > h \end{cases}$$

Where: $l < m < h$ and

- l - the lower limit of the distribution
- h - the upper limit of the distribution
- m - the mode of the distribution.

1. Bidding for Contract Games with Two Bidders

Assume that two bidders are competing for a contract and the bidders' production cost c_i has a triangular distribution over the range [0, 2] with mode value [1].

The assumed production cost has a triangular distribution with probability density function f(c) is given by:

$$f(c) = \begin{cases} c & 0 < c < 1 \\ 2 - c & 1 < c < 2 \\ 0 & \text{elsewhere} \end{cases}$$

The commulative distribution function F(c) of the production cost is given by:

$$F(c) = \begin{cases} 0 & c < 0 \\ c^2 & 0 < c < 1 \\ 1 - (2 - c)^2 / 2 & 1 < c < 2 \\ 1 & c > 2 \end{cases}$$

The triangular distribution has a special characteristic, i.e. it has two distributions over the interval of [0, 2]. The dividing limit of the interval is the mode value of the distribution.

Definition of the bidding function for cost interval [0, 1]

$$E(\pi) = (b - c)*(1 - F(c)) = (b - c)*(1 - b^2/2)$$

The bidders' expected profit $E(\pi)$ is maximized if the $dE(\pi)/db = 0$

$$dE(\pi)/db = (1 - b^2/2) + (b - c)*(-b) = 0$$

Computing the formula:

$$1 - b^2/2 - b^2 + b*c = -3* b^2/2 + b*c + 1 = 0$$

Applying the Quadratic Formula:

$$b = (c + (c^2 + 6)^{0.5})/3$$

This is the bidding function for players over the interval [0, 1] of the triangular distribution

Definition of the bidding function for cost interval [1, 2]

$E(\pi) = (b - c)*(2 - b)^2/2$ is the bidders' expected profit $E(\pi)$.The bidders' expected profit is maximized if $dE(\pi)/db = 0$.

$$dE(\pi)/db = 1/2*((2 - b)^2 + 2*(b - c)* (2 - b)*(-1)) = 0$$

$$(2 - b)*(2 - b - 2*b + 2*c) = 0$$
$$b = (2 + 2*c)/3$$

This is the players' bidding function over interval [1, 2] of the triangular distribution. This Thesis will use the derived formulas to simulate the equilibrium strategy of the bidders with a triangular cost distribution.

2. Bidding for Contract Games with Three Bidders

Assume that three bidders are competing for a contract and the bidders' production cost c_i has a triangular distribution over the range [0, 2] with mode [1].

Definition of the bidding function for cost interval [0, 1]

$$E(\pi) = (b - c)*(1 - F(c))^2 = (b - c)*((1 - b^2/2))^2$$

The bidders' expected profit $E(\pi)$, is maximized if $dE(\pi)/db = 0$

$$dE(\pi)/db = ((1 - b^2/2))^2 + (b - c)*2*(1 - b^2/2)*(-b) = 0$$
$$(1 - b^2/2)*(1 - b^2/2 - 2* (b - c)) = 0$$

Which is true if $-5* b^2/2 + 2*b*c +1 = 0$

Applying the Quadratic Formula:

$$b = [2*c + (4*c^2 + 10)^{0.5}]/5$$

This is the players' bidding function for the interval [0, 1] of the triangular distribution

Definition of the bidding function for cost interval [1, 2]

$$E(\pi) = (b - c)*[(2 - b)^2]^2/2 = [(b - c)*(2 - b)^4]/2$$

The bidders' expected profit $E(\pi)$, is maximized if $dE(\pi)/db = 0$

$$d E(\pi)/db = (2 - b)^4 + 4*(b - c)* (2 - b)^3*(-1) = 0$$
$$(2 - b)^3*(2 - b - 4*b + 4*c) = 0$$
$$b = (2 + 4*c)/5$$

This is the bidding function for the players for the interval of [1, 2] of the triangular distribution. This Thesis will use the derived formulas to simulate the equilibrium strategy of the bidders with triangular cost distribution.

F. SUMMARY

This Chapter has explored auctioning theory. In the first part of the Chapter, the class of the games to which auctioning activity belongs were analyzed and defined as non-cooperative games with incomplete information. The second part of this Chapter analyzed the bidding for contract game. The third part of this Chapter summarized the basics of order statistics, which will later be used to control the simulation outcome. The theoretical description of the game provided the opportunity to find the bidders' equilibrium strategies.

This thesis assumes that all the bidding firms are equally efficient; their cost distributions are identical. Further it is assumed, that the bidders' costs distribution is either uniform or triangular over an interval of [l, h]. The distribution is known to all bidders. These assumptions were used to define the equilibrium bidding function. After defining the normal-form representation of the bidding for contract game, this Chapter derived the equilibrium bidding function for the those games that analyzed here. The bidders are expected to submit bids that maximize their profit; these bids form a Bayesian Nash equilibrium.

III. SIMULATION OF THE FPSBA

This Chapter verifies the results derived in the previous Chapters using computer simulations. The first section of the Chapter characterizes the model used to simulate the FPSBA processes. The second part of the Chapter summarizes the simulation results and the insights gained from simulating the bidding for contract games. The first simulations assume that the production cost has a uniform distribution; the second simulations assume that the production cost satisfies a triangular distribution. In concluding, the Chapter summarizes the FPSBA simulation findings.

A. THE FPSBA MODEL

This section of the thesis will develop a FPSBA model for simulating the bidding for contract game. After analyzing bidder behavior and using the findings from Chapter II, this section provides a mathematical model of FPSBA. The mathematical model will be transformed into computer code to conduct the simulations.

1. Simulation of Processes in FPSBA

Simulation imitates real-world phenomenon, processes or systems. Simulation generates an artificial history of a system, and observes that artificial history to make judgments concerning the operating characteristics of the real-world system.

> A model is defined as a representation of a system for the purposes of studying the system. For most studies, it is not necessary to consider all the details of a system; thus a model is not only a substitute for the system, it is also a simplification of the system. On the other hand the model should be sufficiently detailed to permit valid conclusions to be drawn about the real system. [Ref. 14]

By developing a simulation model we can study the processes and behavior of the system, and its changes over time. To model a system it is necessary to understand the system's concepts and boundaries. A system is often affected by the environment in which the system is operating. These variables are considered to be the system environment.

Models can be classified as being mathematical or physical. A mathematical model uses symbolic notation and mathematical equations to simulate a system. Mathematical models can be classified as deterministic or stochastic simulation models. A stochastic model uses one or more random variables as inputs. These random inputs generate random outputs. Since the outputs of the model are random, stochastic models can imitate the real system. However, the simulation results must be treated as a statistical estimate of the real-world system's characteristics.

In the case of FPSBA, the model represents the bidders and auctioneers. It includes the rules and regulations of the auctions, the bidders' behavior, and their attitudes. The actual and expected market conditions, the availability of resources and other factors are also an influential part of the FPSBA system. The elaborated FPSBA system model incorporates some of these variables. However, to keep the model manageable, a number of simplifications have been made.

2. Information Space of the Game and Strategic Behavior of the Bidder

This section will analyze one of the most important aspects of the model, the bidders' and the buyer's characteristics. The bidding environment will be analyzed as well. Because of the wide variety in bidding regulations, this thesis assumes regulations are met both by the bidders and the buyer. Both parties comply with the applicable law at the time of the auction.

a. Information Space of the Auction

We assume that the FPSBA is a non-cooperative game in which the players have limited information. However, the players know the following:

1. The buyer is fully committed not to deviate from the FPSBA rules during the auction process, even if the deviation is in the buyer's interest. The rules of the auctions are common knowledge.

2. The bidders' utility function is defined by a von Neuman -- Morgenstein utility function U(·), and it is common for all bidders.
3. The bidders know their production cost (c_i) with certainty when they bid and this cost is private information known only to the bidder. However, the bidders have subjective assumptions about the range and distribution of production costs for other bidders. This Thesis assumes that the probability distribution for each bidder is the same; it follows either a uniform or a triangular probability distribution over the production cost range.
4. The bidders know with certainty the number of participants submitting bids.
5. While preparing for auctions, the bidders send signals, sometimes misleading, about their cost to other bidders. However, cooperation between the bidders is restricted.

b. The Players' Strategic Behavior

A number of assumptions about the bidders will be made to construct a comprehensive FPSBA model. The buyer and the bidder are expected to act rationally. The rationality of the bidders means that:

1. The bidders pursue their own self interest; they attempt to maximize their profit from the auction. Bidders maximize profits by submitting the highest possible bid. However, the bidders recognize that they are constrained by the other participants' bids. The higher the bid, the lower the probability of winning the auction. This self-regulating mechanism provides an efficient solution for the game.
2. Bidders consider their production costs and the production costs of the other bidders. The resulting bids form a Bayesian Nash equilibrium. Bidders using the equilibrium strategy simultaneously maximize the expected profit regarding both their and other participants' expected bids.

3. The buyer and the bidders are risk neutral. The risk neutrality assumption is disputable. This thesis assumes risk neutrality to simplify the model formulation, the calculations and the description of the bidders' strategic behavior.

3. Model Description

Consider a competitive bidding model in which the buyer announces a contract to procure a specified commodity. This Thesis assumes that there are n bidders for a particular procurement and they are responsive and responsible. It is assumed that bids are solicited and the contract awarded to the lowest bidder; the bids differ only in price. The contract specifies the winning firm's total receipts from the buyer. The winner's expected profit depends both on the bid submitted and the cost incurred. In turn, the bids are influenced by the firm's expectation about the competing bids.

The constructed model attempts to capture the major and decisive characteristics of the real FPSBA process. However, this model is only a first approximation of many procurement procedures. The model provides an opportunity to experiment with the bidders' possible actions and decisions during FPSBA. The proposed FPSBA model assumes symmetry of information and preferences, which makes it possible to concentrate exclusively on a symmetric Bayesian Nash equilibrium.

4. The Computer Simulation Methodology

This thesis used a personal computer and Excel 5.0 spreadsheet software to simulate the FPSBA process. The computer simulation flowchart is in Figure 2. A computer program was written to conduct the necessary operations to simulate the process. The simulation results were collected in a separate worksheet. The composed computer program had to be adjusted during the simulation to get the desired number of auctions.

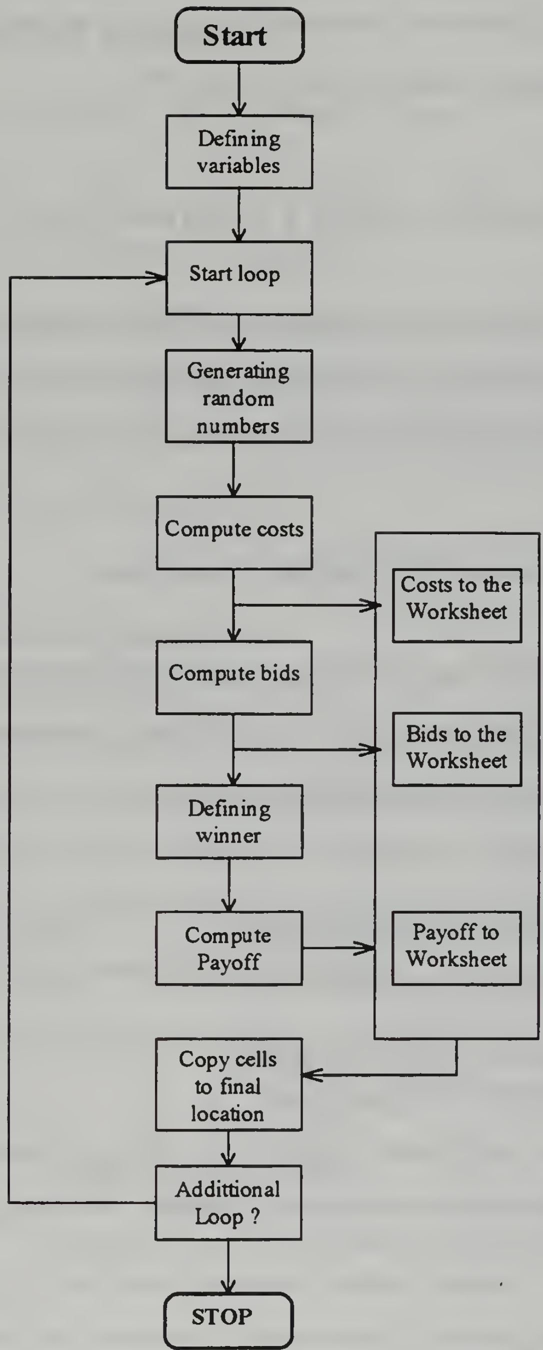

Figure 2 The computer simulation flowchart

The bidder's initial production cost, which was a random variable during the entire simulation process, was generated by Excel's built-in random number generator. The built

in random number generator provides a random number distributed uniformly over the interval [0 , 1]. If it was necessary, the generated random number was transformed using the Inverse Transformation Technique. [Ref. 14:pg. 299]

B. EXPERIMENTATION UNDER A UNIFORM COST DISTRIBUTION

This section of the Chapter describes the FPSBA simulation when the production costs are distributed according to the uniform distribution. The first part of the section explains the mathematical simulation model; the later part presents the simulation results.

1. The Mathematical Model for Simulation

Consider a situation where all firms are equally efficient and their cost distributions are identical. In particular, assume that the bidders' potential production costs are distributed uniformly over an interval [h, l].

For this case:

- the probability density function of cost is $f(c) = 1/(h - l)$
- the probability distribution function of cost is $F(c) = (c - a)/(h - l)$

Where: h - upper limit of the cost range
l - lower limit of the cost range

Determining the equilibrium strategies requires simultaneously analyzing all the bidding decisions. The model incorporates Chapter II's findings about the bidders' equilibrium strategy.

A bidding strategy defines the relationship between the bidder's proposal, b_i, and cost c_i. Appendix A derives the bidding functions for the simulated game. The bidding functions take the general form:

$$b_i = ((h - l) + (n - 1)*c_i)/n$$

With two bidders, the bidding function takes the form:

$$b_1 = (1 + c_1)/2 \qquad \text{and} \qquad b_2 = (1 + c_2)/2$$

for Bidder 1 and Bidder 2, respectively.

With three bidders, the bidding function takes the form:

$$b_1 = (1 + 2^*c_1)/3 \qquad \text{and} \qquad b_2 = (1 + 2^*c_2)/3 \qquad \text{and} \qquad b_3 = (1 + 2^*c_3)/3$$

for Bidder 1, Bidder 2 and Bidder 3, respectively.

These formulas mathematically articulate the bidders' behavior. They reflect the bidders' expected bidding decision described in the previous section. This thesis assumes that the bidder's production cost was drawn from the stated distribution. The bidders' cost was simulated with a random number generator.

2. Random Number Generation

This thesis applied the Built-in Random Number Generator of the Excel 5.0 spreadsheet software to generate uniformly distributed random numbers over the required interval. The built-in random number generator provides with random number over the interval [0, 1]. The simulations often use different cost distribution intervals. Therefor, the generated random numbers were transformed as necessary.

The required random number transformation was based on:

$$R = (h - l)^*\text{Rand}() + l$$

Where:
- R - required random number
- l - lower limit of the required random number
- h - upper limit of the required random number
- Rand() - the Excel generated random number

3. Experimental Setting for FPSBA with Two Bidders

This section assumes that two bidders are bidding for a contract, and the lowest bid wins. The bidder's production cost is distributed between [0, 1] according to the uniform distribution. The generated cost numbers are analyzed. This Thesis used the Excel

built-in statistical analyses program package to examine the quality of random numbers used during the simulations. The analysis result are shown in Appendix E.

During the simulation, three different scenarios were assumed and analyzed. In the first scenario, both bidders used equilibrium strategies. In the second scenario, one of the bidders used the equilibrium strategy while the other bidder used non-equilibrium strategy. In the third scenario, both of the bidders used a non-equilibrium strategy.

The different scenarios were designed to illustrate the theoretical finding that the equilibrium bidding strategy of the bidders is a strategy that maximizes the bidders' payoff. Any deviation from the equilibrium strategy would reduce expected profit for the deviating bidder. The numerical simulation results are shown in the Appendix C.

a. *Graphic Presentation of the Computer Simulations Results*

The simulation was used to find an equilibrium in the bidding for contract game. The bidders' average expected profit can demonstrate the presence of an equilibrium strategy. If the bidders achieve equal average profit in a series of auctions conducted under the same conditions, this outcome represents the equilibrium of the game. Simulation results are shown in Figure 3.

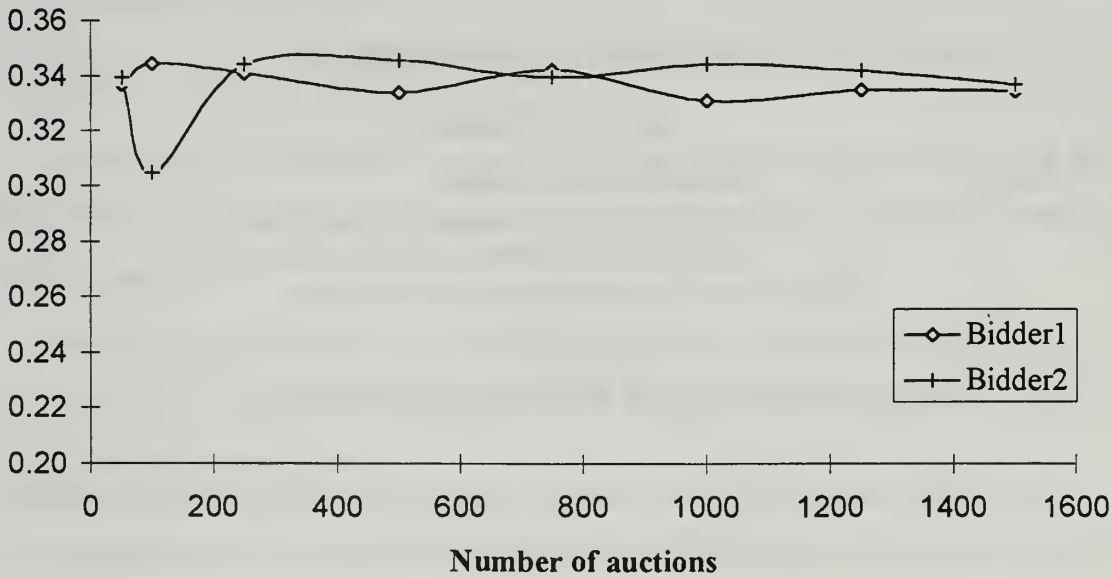

Figure 3 Expected profit from winning the auction

Figure 3 shows there exits a well-defined equilibrium in these games. The winning bidders' expected profit if they win the auction approaches 0.334 as the number of simulated auctions approaches 1500. The simulation showed that the bidders won equally, and their expected profit approached to the analytical computed result 0.3333. The computation is shown in Appendix B.

The average profit per auction considers all auctions in which the bidder participates. It better characterizes the bidders' expectations and their motivations. The bidder seeks to maximize the average profit per auctions not the profit per auction won. In the following simulations, this Thesis used the average profit from bidding to analyze the outcomes of the different bidding strategies. The average expected profit graph, shown on Figure 4, reinforces the existence of a well-defined equilibrium in these games. The bidders' average profit approaches 0.167 as the number of simulated auctions approaches 1500.

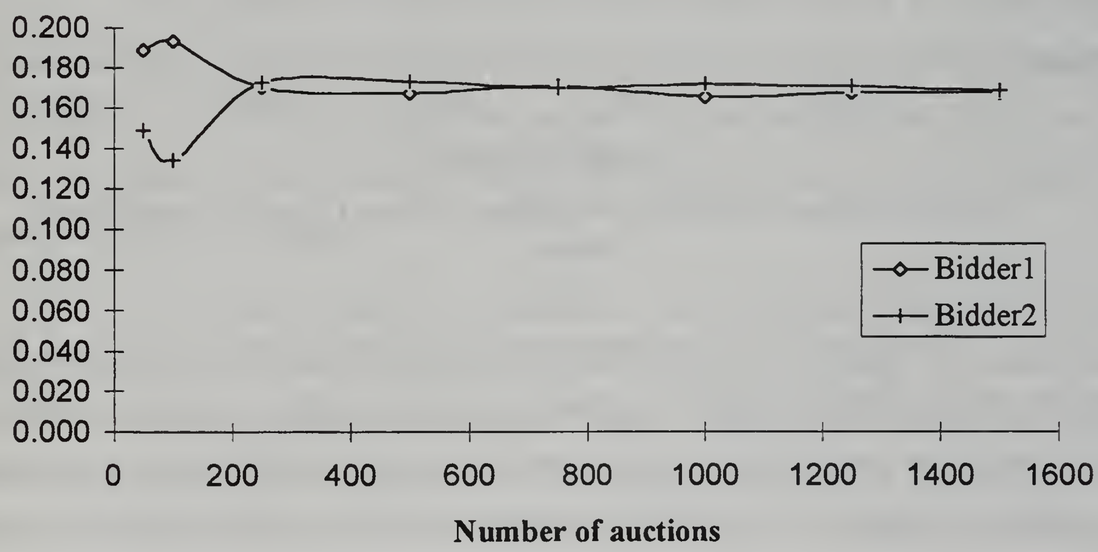

Figure 4. Average profit from bidding -- both bidders using equilibrium strategy

An equilibrium strategy should also simultaneously maximize both bidders' profits. If a strategy maximizes the bidders' payoff, no-bidder is willing to deviate from the

strategy. This is an equilibrium of the game. This equilibrium condition was explored in the next simulation. The simulation results are shown in Figure 5.

In this simulation, one of the bidders -- Bidder 1 -- used a non-equilibrium strategy. This simulation assumed that Bidder 1's bid was 0.1 higher than the non-equilibrium strategy. The bidding function for the bidder was

$$b_1 = [(1 + c_1)/2] + 0.1$$

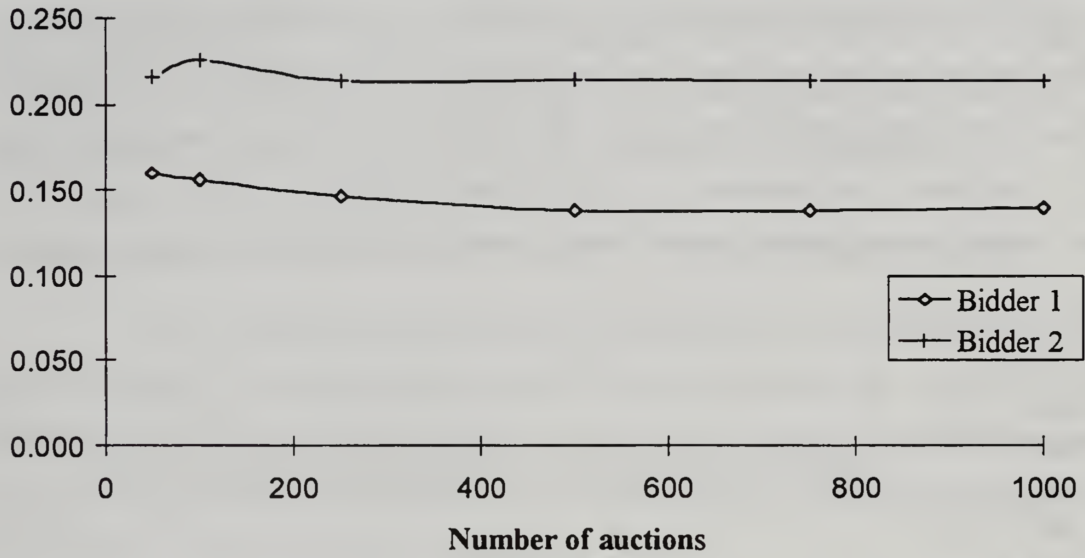

Figure 5 Average profit from the bidding -- bidder 1 using non- equilibrium strategy

The simulation revealed that the deviant bidder's profit was lower than the profit using the equilibrium strategy. The bidder playing the equilibrium strategy -- Bidder 2 -- received higher profits in this game compared to the game in which both players used the equilibrium strategy. To control the simulation result, an other simulation was conducted with different non-equilibrium strategy.

In this case, Bidder 2 used a non-equilibrium strategy described with bidding function:

$$b_2 = [(1 + c_2)/2] - 0.1$$

Simulation results are shown in Figure 6.

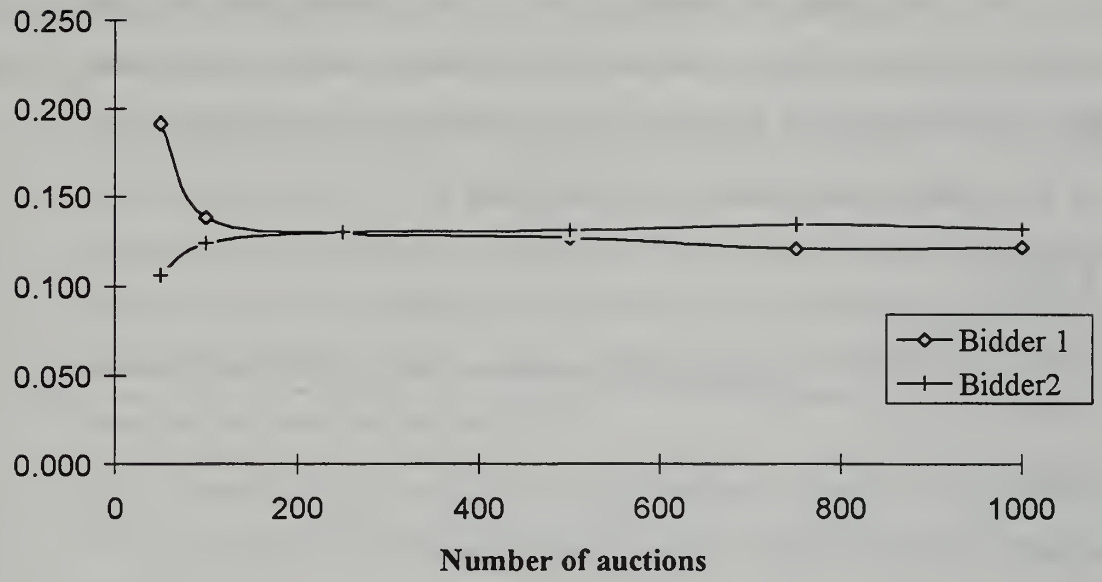

Figure 6 Average profit from the bidding -- Bidder 2 using a non - equilibrium strategy

The simulation revealed that the deviant bidder's profit was lower after deviating from the equilibrium strategy. However, in this situation the equilibrium strategy bidder did not achieve higher profit. The deviant 'low" bids made Bidder 2 win more often than Bidder 1. This reduced the profits for both of the bidders, even if Bidder 1 played the equilibrium strategy.

To further support the existence of an equilibrium strategy, the next scenario assumed both bidders deviated from their equilibrium strategy. The simulation was used to ascertain if either could achieve a higher average payoff in this case. This simulation assumed that the deviation from equilibrium is random, as opposed to the previous two cases when the deviant bidder used systematic non-equilibrium strategies. The bidding function is given by:

$$b_i = (1 + c_i)/2 \ (+ \text{ or } - M_i)$$

Where: M_i - Random number between [0, 0.5] for Bidder 1 and [0, 0.3] for Bidder 2

The addition or subtraction of the random number depended on the value of random number M_i. If M_i was less than 0.3 for Bidder 1, and it was less than 0.1 for Bidder 2, than M_i was added otherwise it was subtracted from the equilibrium bid.

Simulation results are shown in Figure 7.

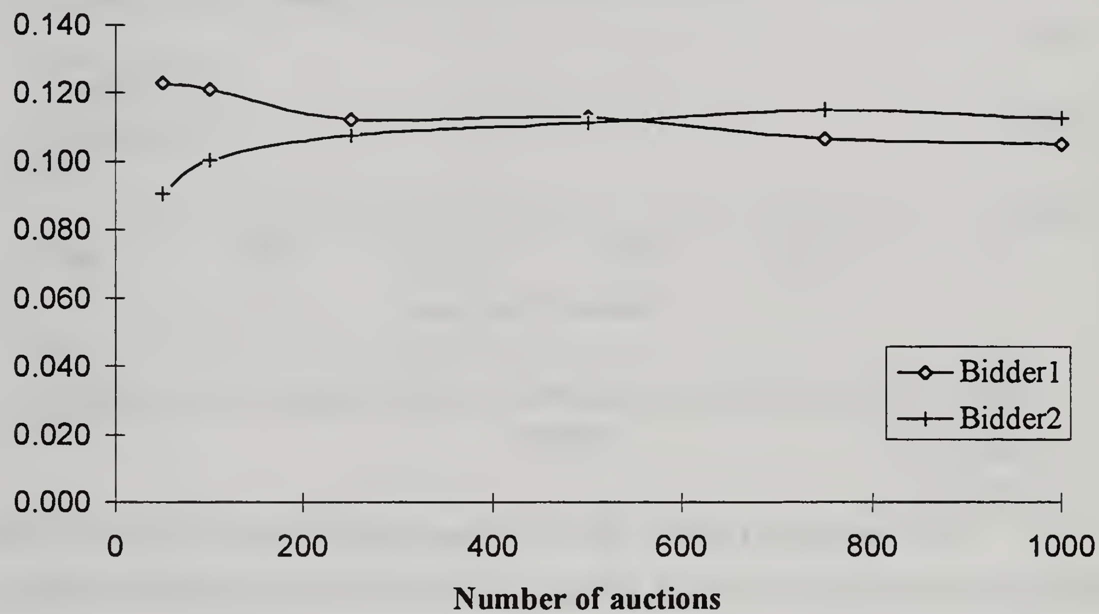

Figure 7 Average profit from bidding -- both of the bidders using a non-equilibrium strategy

This simulation shows that the average profit curves do not follow any defined pattern, though they intersect at one point. Figure 7 also demonstrates that both of the bidders had lower payoffs in this case compare to the two previous simulations.

b. Summary

After analyzing the results of the two-bidder computer simulation, it can be claimed that:

1. The simulation of the bidding for contract game is valid. The computation completed in Appendix B shows **0.3333** as an expected difference between the lowest and the highest costs. The expected difference between the highest and the lowest cost is the winner's profit. The simulation found **0.334** as the

winners' average profit after 1500 simulated auctions. Further simulations would approach the theoretical results.computation is shown in Appendix B.

2. The average profit per auction better characterizes the bidders' expectations and their motivations. The bidder seeks to maximize the average profit per auctions not the profit per auction won. The average expected profit graph, shown on Figure 4, reinforces the existence of a well-defined equilibrium in these games. The bidders' average profit approaches 0.167 as the number of simulated auctions approaches 1500.

3. The equilibrium strategy maximizes the bidders' expected average profit; deviations from the equilibrium strategy cause an expected loss for the deviant bidder.

4. If the deviation from the equilibrium strategy is negative, i.e., bids are below the equilibrium bid, both of the bidders have lower expected profits.

5. If the bidder follows equilibrium strategy, and its opponent deviates from equilibrium strategy in a positive direction, the equilibrium strategy bidder's expected profit will be higher on average.

The change in number of competing bidders has a significant impact on the outcomes of the bidding for contract game. The following section of this Chapter will analyze this effect.

4. Experimental Setting for FPSBA with Three Bidders

This simulation was conducted to find the effect on the game equilibrium of changing the number of bidders. The simulation assumed that three participants were bidding for a contract; the lowest bid wins. The bidders' production costs are distributed between [0, 1], according to the uniform distribution.

During the simulation, three different scenarios were analyzed, as in two-bidder simulation case. In the first scenario, all the bidders used the equilibrium strategy. In the

second scenario, two bidders used the equilibrium strategy while the other bidder used a non-equilibrium strategy. In the third scenario all the bidders used non-equilibrium strategies.

The scenarios were designed to support the theoretical finding that the bidders' equilibrium strategy is dominant. In other words, the equilibrium strategy maximizes the payoff to all the bidders. Deviating from the equilibrium strategy reduces the payoff for the deviating bidder. The numerical simulation results are shown in Appendix C.

a. *Graphic Presentation of the Computer Simulation Results*

Figure 8 shows there is a well-defined equilibrium in the three person equilibrium strategy game, as in the two-player game. Figure 8 indicates that the winning bidders' average profit approaches 0.25 as the number of auctions approaches 2500.

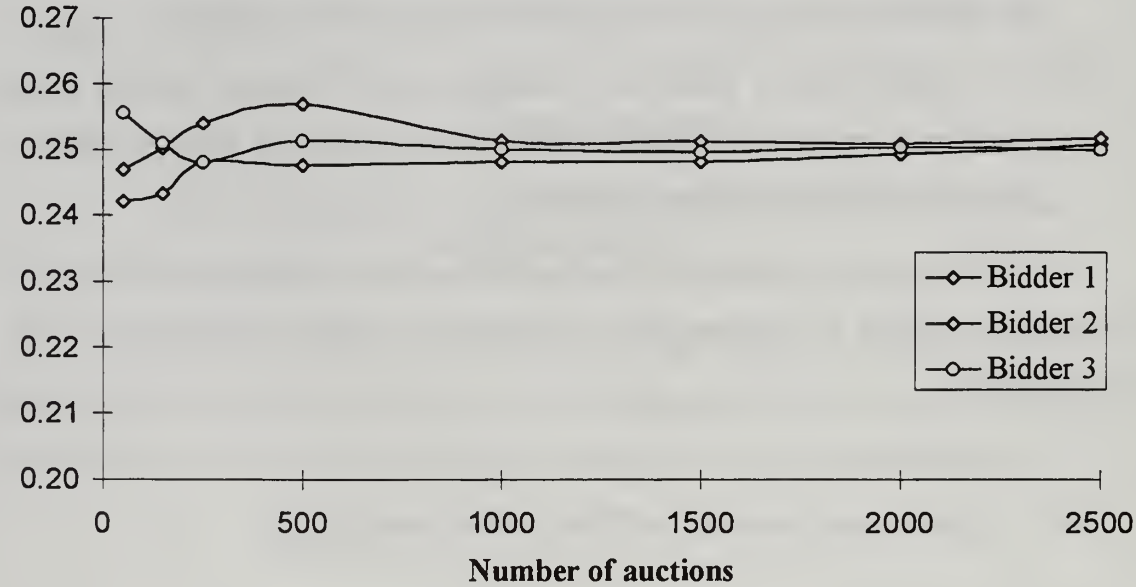

Figure 8 Expected profit from winning the auction

The simulation indicates that increasing the number of bidders requires mores auctions to converge to the equilibrium average profit. Introducing an additional competitor decreased the average expected profit from 0.333 with two bidders to 0.25.

The average profit per auction considers all auctions in which the bidder participates. It is a better characteristics for the bidders expectations and their motivations. The bidder seeks to maximize the average profit per auctions not the profit per auction won. The average expected profit graph is shown in Figure 9.

The average expected profit graph reinforces the existence of a well-defined equilibrium in these games. The bidders' average profit approaches 0.835 as the number of simulated auctions approaches 2500.

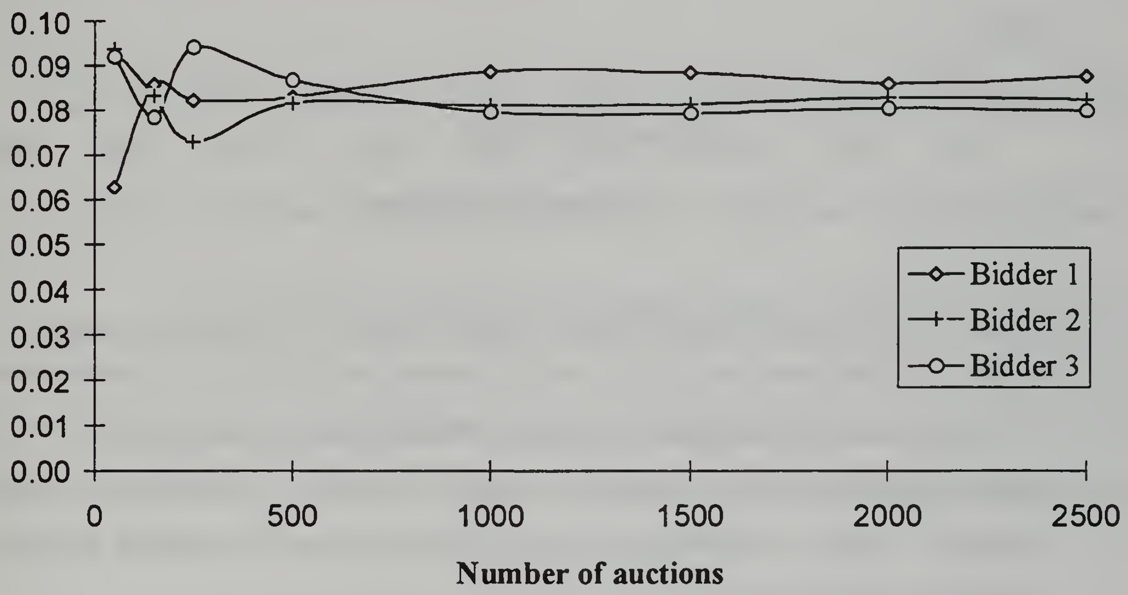

Figure 9 Average profit from bidding -- all of the bidders used equilibrium strategy

An equilibrium strategy should also simultaneously maximize all bidders' profits. If a strategy maximizes the bidders' payoff, no-bidder is willing to deviate from the strategy. This is an equilibrium of the game. This equilibrium condition was explored in the next simulations.

In these simulations, one of the bidders -- Bidder 3 -- used a non-equilibrium strategy. First, Bidder 3's bids 0.2 more than the equilibrium strategy. The bidding function was:

$$b_3 = [(1 + 2*c_3)/3] + 0.2$$

Simulation results are shown in Figure 10.

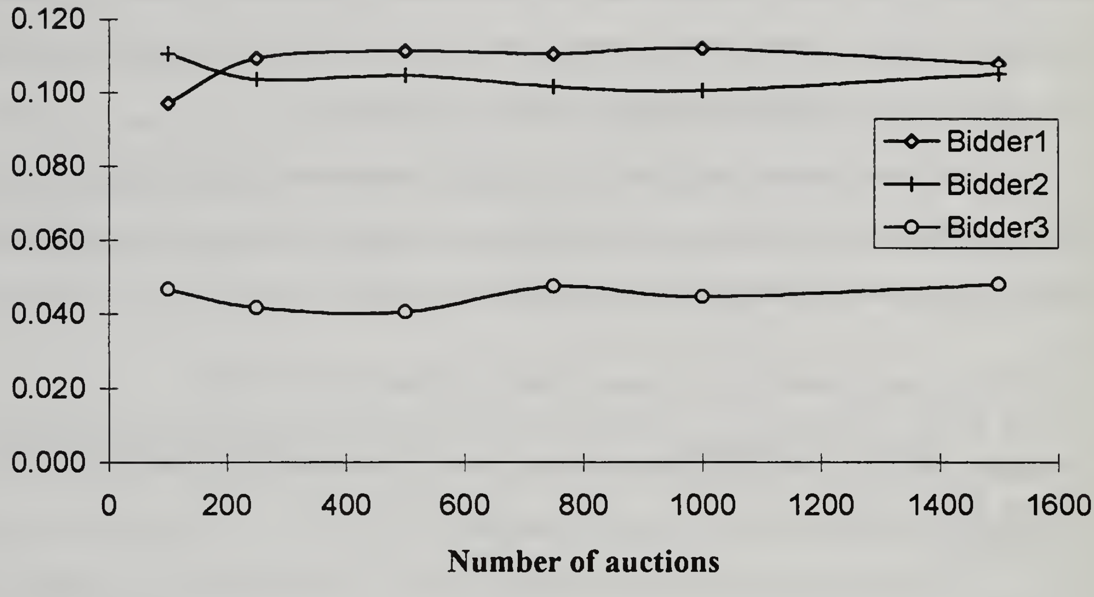

Figure 10 Average profit from bidding -- Bidder 3 using non-equilibrium strategy

Simulation of bidding with one non-equilibrium bidder reveals that:

1. the bidders playing the equilibrium strategy (Bidder 1 and Bidder 2) can achieve a higher average payoff compare to the game where all the players used the equilibrium strategy.

2. the non-equilibrium player receives a lower average profit than under the equilibrium strategy.

To verify the simulation result, a second simulation was conducted with a different non-equilibrium strategy. In this case Bidder 3 used strategy described by bidding function:

$$b_3 = [(1 + 2*c_3)/3] - 0.15$$

Simulation results are shown in Figure 11.

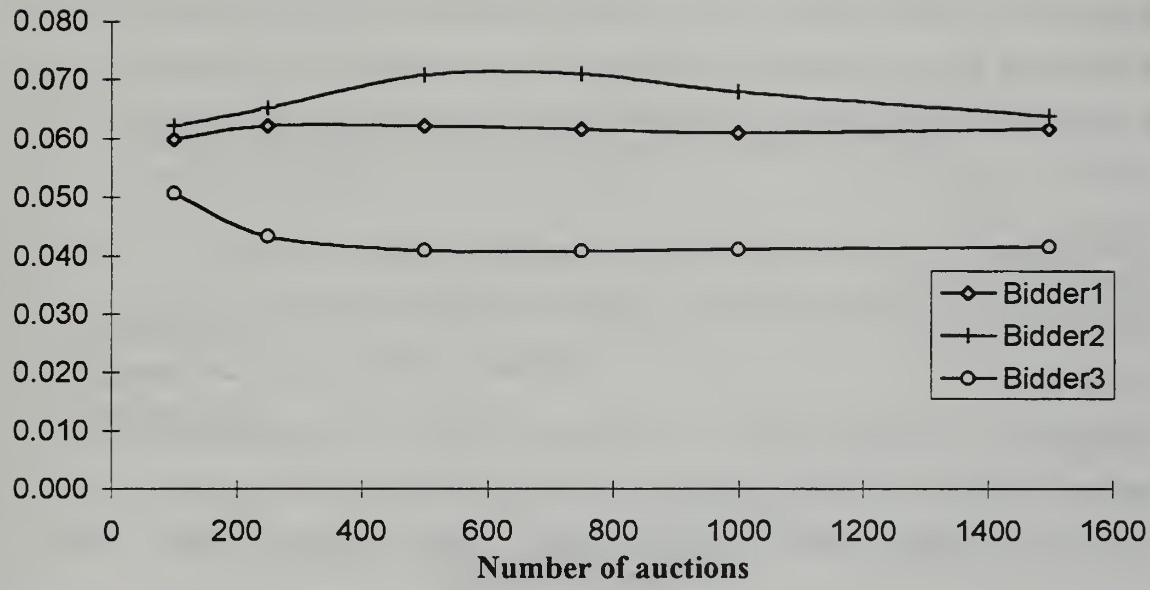

Figure 11 Average profit from bidding, Bidder 3 using a non-equilibrium strategy

The simulation revealed that the deviant bidder's profit was lower in this case compare to the equilibrium strategy. However, the competing bidders in this situation did not achieve higher profits. With the deviant 'low" bids, Bidder 3 won more often than Bidder 1 and Bidder 2; Bidder 3's lower bid and more frequent winning lowered the profits for all bidders, even the equilibrium strategy bidders.

To further support the existence of an equilibrium strategy, the next scenario assumed all bidders deviate from their equilibrium strategy. The simulation was used to ascertain if any bidders could achieve higher average payoff in this case. This simulation assumed a random deviation from the equilibrium. The bidding function for all bidders is:

$$b_i = (1 + 2*c_i)/(3 + 0.6*\text{Rand}()) \qquad i = 1, 2, 3$$

Simulation results are shown in Figure 12. This simulation shows that the average profit curves follow a defined pattern. The random but symmetric bidding strategy generated this outcome.

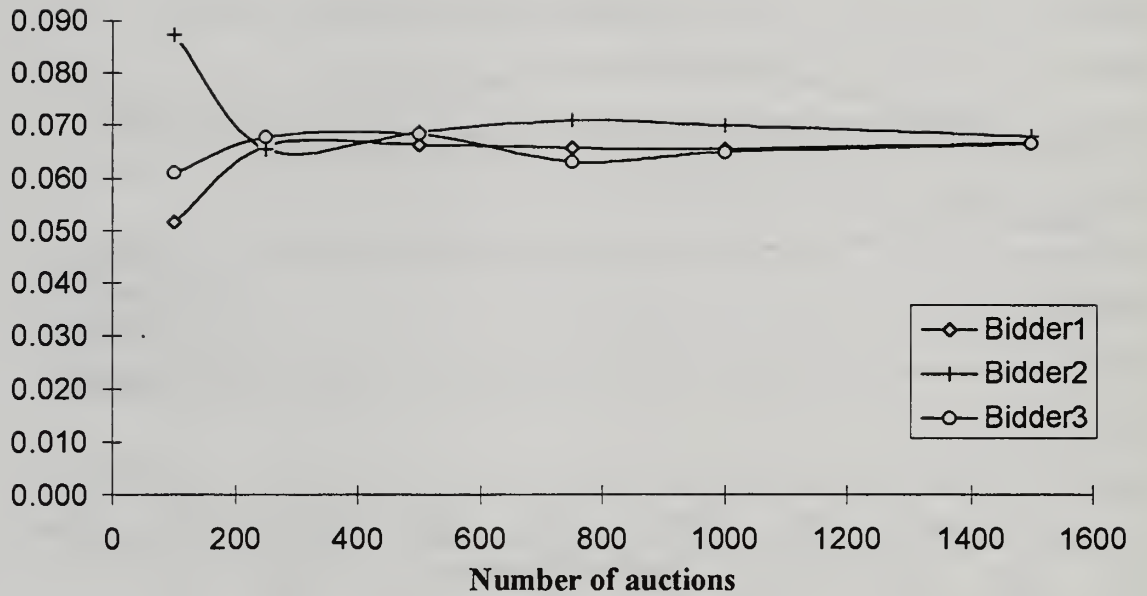

Figure 12 Average profit from bidding --all bidders using non-equilibrium strategy

If the bidders followed a non-symmetric random deviation from their equilibrium strategy, the expected average profit curve not have a consistent pattern. Figure 12 also demonstrates that all bidders had lower payoffs in this case compared to the equilibrium strategy simulation.

b. Summary

After analyzing the results of the three-bidder computer simulation, it can be asserted that:

1. The simulation showed an average payoff of winning approaches 0.25 after 2000 simulated auctions. This result confirms the theoretical prediction that number of bidders in the bidding for contract games is one of the decisive elements. The higher the number of the bidders, the lower the bidders' expected profit. The theoretical calculation of average expected profit in the three-bidder game is shown in Appendix B. The computed value of the first and second lowest costs differences gives the auction winner's expected average profit . The simulated expected profit was 0.2507; further simulations

would approach the theoretical result. The average expected profit graph reinforces the existence of a well-defined equilibrium in these games. The bidders' average profit approaches 0.0835 as the number of simulated auctions approaches 2500.

2. The equilibrium strategy maximizes the bidders' expected average profit; deviations from the equilibrium strategy cause an expected loss for the deviant bidder, as in two-bidder simulation.
3. If the deviation in bidding is negative, i.e. bids are below the equilibrium strategy, then all bidders had lower expected payoffs. If one bidder deviates from equilibrium strategy in positive direction, than the expected profit of the equilibrium strategy will be higher on average.

5. End Result of the Simulations with Uniform Distribution

After analyzing the computer simulations for the uniform cost distribution, it can be observed that:

1. The conducted simulations validate the constructed FPSBA model. The simulation results approached the theoretical results indicated by the order statistics computed for the two and three bidders cases. The simulation results converge to the theoretical results if sufficient auctions are simulated.
2. The equilibrium strategy maximizes the bidders' expected average profit; deviating from the equilibrium strategy reduces profits for the deviant bidder. The non-equilibrium strategy reduces average profits for all the bidders if the deviation is below the equilibrium bid.
3. Adding a new competitor to the game does not change the nature of the game; the additional competitor lowers the expected bid and the average expected profit.

C. EXPERIMENTATION UNDER A TRIANGULAR COST DISTRIBUTION

The FPSBA simulation using a triangular production cost distribution defines the bidders' behavior under a different cost condition. The uniform production cost distribution was used primarily for simplicity, though it is reasonable when there is no ex-ante information about the bidders' production costs. The triangular distribution may be a better approximation of reality. By manipulating the triangular distribution's mean value, average deviation, and minimum and maximum values, one can obtain a first order approximation of the bidders' production cost distribution.

1. The Mathematical Model for Simulation

Consider a situation where all firms are equally efficient and their probability distributions of cost are identical. In particular, assume bidders' costs are distributed according to the same triangular probability distribution. The range of their production cost is defined over the interval [l, h]; it has a mode value m with probability 2/(h - l). For the general case, suppose that the bidders' costs have been distributed according to triangular distribution with parameters:

1. Probability density function:

$$f(c) = \begin{cases} 2*(c - l)/(h - l)*(m - l) & l < c < m \\ 2*(h - c)/(h - l)*(h - m) & m < c < h \\ 0 & \text{elsewhere} \end{cases}$$

2. The cumulative distribution function:

$$F(c) = \begin{cases} 0 & c < l \\ (c - l)^2 /(h - l)*(m - l) & l < c < m \\ 1- (h - c)^2 /(h - l)*(h - m) & m < c < h \\ 1 & c > h \end{cases}$$

where: l - lower limit of the bidders' cost distribution
h - upper limit of the bidders' cost distribution
m - the mode of the distribution (in extreme l = m or h = m

A bidding strategy defines the a relationship between the bidder's proposal, b_i and cost c_i. The equilibrium bidding functions with a triangular distribution takes the general form:

1. For the production cost interval $l < c < m$

$$b = \frac{n*l + (n-1)*c + \{(n*l + (n-1)*c)^2 + 2*[((n-1) + 0.5)*(K1 - l^2 - 2*(n-1)*c*l)]\}^{0.5}}{2*[(n-1) + 0.5]}$$

Where: $K1 = (h - l)*(m - l)$

2. For the production cost interval of $l < c < m$

$$b = \frac{h + 2*(n-1)*c}{2*(n-1) + 1}$$

The derivation of the general bidding function can be found in Appendix A.

In deriving the bidding functions for the computer simulation, it was assumed that: $l = 0$, $h = 2$, $m = 1$, the simulation assumed that there was either two or three competitors. Substituting these assumptions into the general equilibrium bidding function yields:

1. Two participant equilibrium bidding strategy:

$$b_i = \frac{c_i + (c_i^2 + 6)^{0.5}}{3} \qquad \text{if} \quad 0 < c_i < 1$$

$$b_i = \frac{2 + 2*c_i}{3} \qquad \text{if} \quad 1 < c_i < 2$$

2. Three participant equilibrium bidding strategy:

$$b_i = \frac{2c_i + (4*c_i^2 + 10)^{0.5}}{5} \qquad \text{if} \quad 0 < c_i < 1$$

$$b_i = \frac{2 + 4*c_i}{5} \qquad \text{if} \quad 1 < c_i < 2$$

2. Random Number Generation

The Excel 5.0 built in random number generator provides random numbers. The random numbers are uniformly distributed over the interval [0, 1]. The following simulations need random numbers distributed according to a triangular distribution. Uniformly distributed random numbers can be converted to a triangular distribution using inverse transformation technique. [Ref. 14: pg. 300]

According to this method:

$$c = l + (R^*(m - l)^*(h - l))^{0.5} \qquad \text{for } l < c < m \text{ and}$$

$$c = h - (R^*(h - m)^*(h - l))^{0.5} \qquad \text{for } m < c < h$$

where: c - triangularly distributed random variables

R - uniformly distributed random number generated by Excel 5.0

The generated cost numbers were analyzed. This Thesis used the Excel built-in statistical analyses program package to examine the quality of random numbers used during the simulations. The analytical result are shown in Appendix E.

3. Setting for the Computer Simulation of FPSBA of Two Bidders

Computer simulation of the FPSBA was conducted using an Excel spreadsheet. This setting assumes two competing bidders; the lowest bid wins. The bidders' production cost is distributed between [0, 2] according to the triangular distribution with mode value [1]. The constructed computer program, written as an adjustable macro in Visual Basic language, followed the mathematical model of the bidding for contract game. The applied macro is shown in Appendix D. The macro devised bids for each participant and collected the auction results in a separate worksheet.

During the simulation, three different scenarios were analyzed. In the first scenario, both bidders used the equilibrium strategy. The second scenario assumed that one of the bidders used an equilibrium strategy, while the other used a non-equilibrium strategy. In the third scenario both of the bidders used non-equilibrium strategies.

The different scenarios show the that the equilibrium bidding strategy is a dominant strategy. In other words, the equilibrium strategy yields the maximum payoff for both of the bidders. Any deviation from the equilibrium strategy reduces profit for the deviant bidder. The numerical simulation results are shown in Appendix C.

a. *Graphic Presentation of the Computer Simulation Results*

The simulation was used to find an equilibrium in the bidding for contract game. The bidders' average expected profit demonstrates the presence of an equilibrium strategy. An equilibrium of the game occurs when the bidders achieve equal average profits in a series of auctions conducted under the same conditions.

Figure 13 shows there exits a well-defined equilibrium in these games. The bidders' average profit approaches 0.196 as the number of simulated auctions approaches 1500.

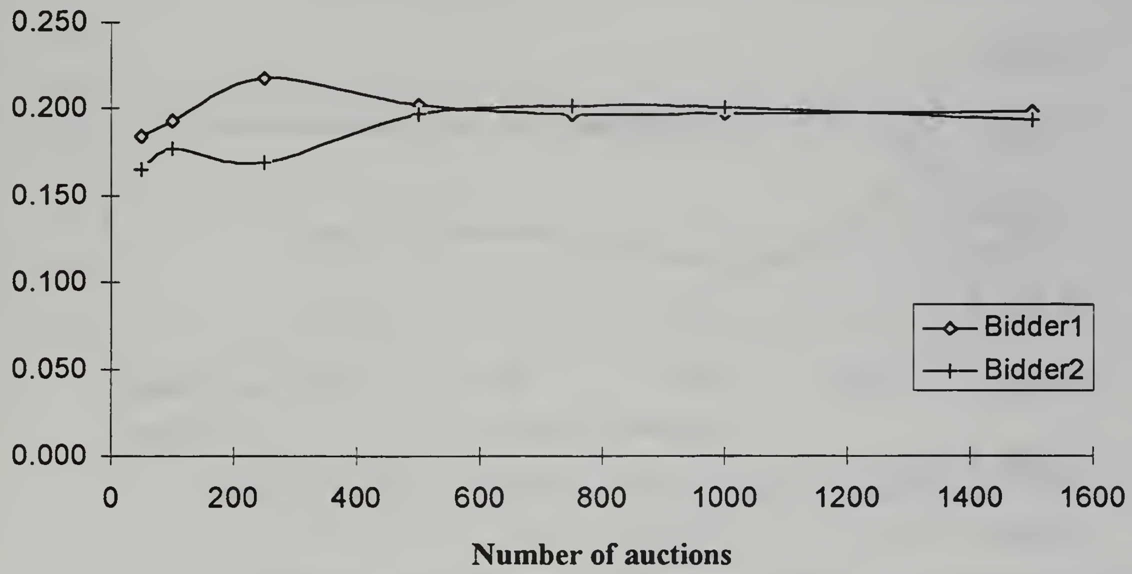

Figure 13 Average profit from bidding -- bidders used equilibrium strategy

An equilibrium strategy should also maximize both bidders' profits. If a strategy maximizes the bidders' payoff, no-bidder is willing to deviate from the strategy.

This is an equilibrium of the game. This equilibrium condition was explored in the following simulations. In this simulation, one of the bidders used a non-equilibrium strategy. In particular, the bid was 0.25 above the equilibrium strategy. In particular, the Bidder 1's bidding function is given by:

$$b_1 = \frac{c_1 + (c_1^2 + 6)^{0.5}}{3} + 0.25 \qquad \text{if} \qquad 0 < c_i < 1$$

$$b_1 = \frac{2 + 2*c_1}{3} + 0.25 \qquad \text{if} \qquad 1 < c_i < 2$$

The simulation revealed that the deviant bidder's average profit was lower compared to the equilibrium strategy. The bidder playing the equilibrium strategy -- Bidder2 -- received a higher profit in this game compared to the case where both players used the equilibrium strategy. Simulation results are shown in Figure 14.

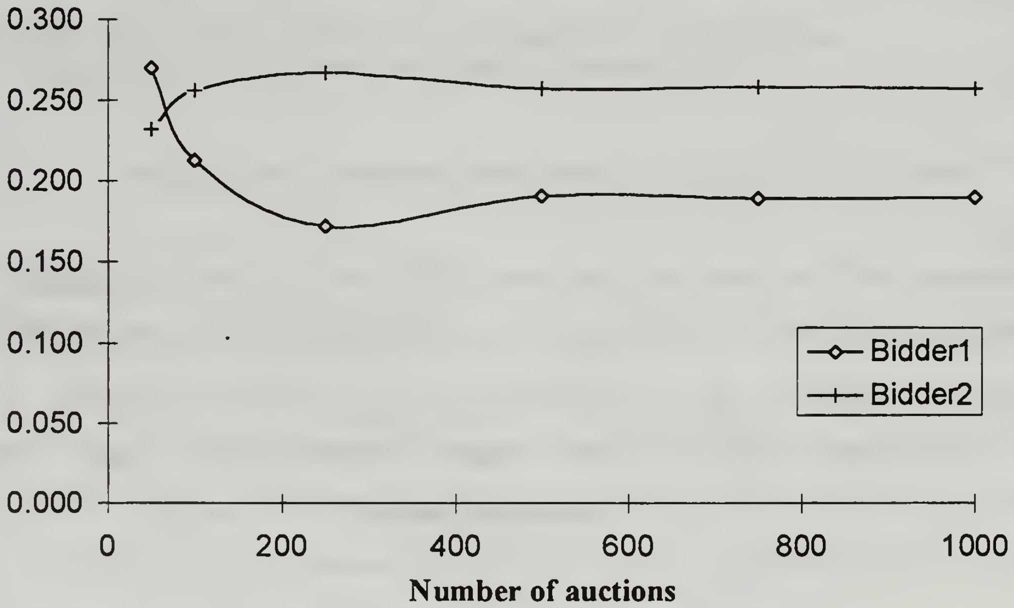

Figure 14 Average profit from bidding -- Bidder 1 used non-equilibrium strategy

To reinforce this simulation result, another simulation was conducted with a different non-equilibrium strategy.

In this case, Bidder 1 used non-equilibrium strategy described with bidding function:

$$b_1 = \frac{c_1 + (c_1^2 + 6)^{0.5}}{3} - 0.25 \qquad \text{if} \qquad 0 < c_i < 1$$

$$b_1 = \frac{2 + 2c_1}{3} - 0.25 \qquad \text{if} \qquad 1 < c_i < 2$$

This scenario assumes that Bidder 1 bids below the equilibrium bid. As a result, Bidder 1 will win more contract, but earn a lower expected profit from the contract. Simulation results are shown in Figure 15.

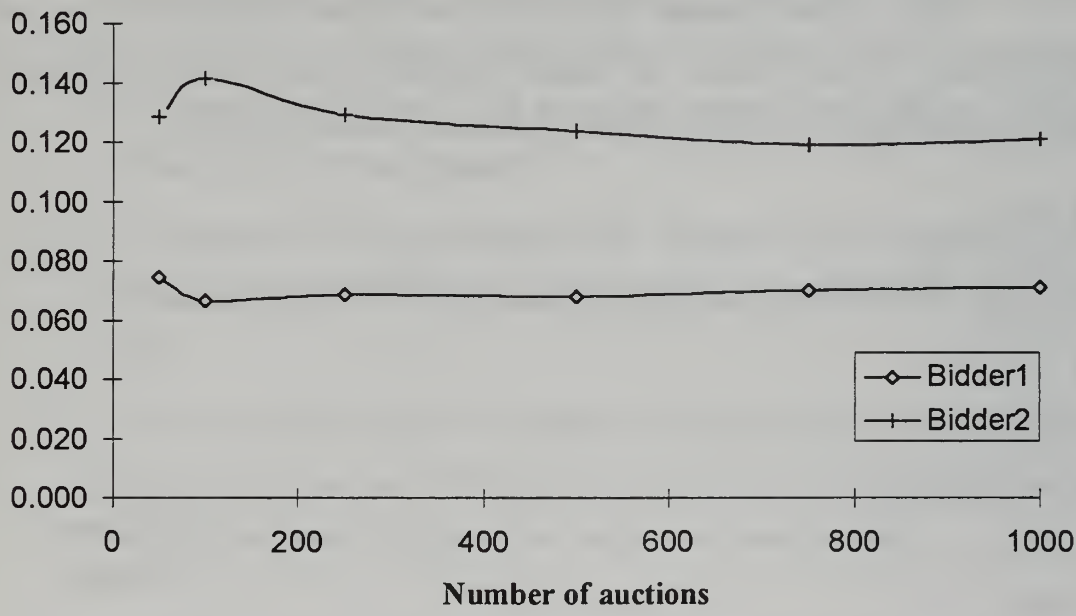

Figure 15 Average profit from bidding -- Bidder 1 used non-equilibrium strategy

The simulation revealed that the deviant bidder's profit was lower compared to the equilibrium strategy. In this situation, the equilibrium strategy bidder also received a lower profit compared to the case where both of the bidders used the equilibrium bidding strategy. The deviant "low" bids allowed Bidder 1 to win more contracts, reducing the equilibrium bidder's expected profit.

To further verify the equilibrium strategy, the next scenario assumes both bidders deviate from the equilibrium strategy. The simulation was used to ascertain if either participant could achieve a higher average payoff in this case. This simulation assumed that the deviation from equilibrium is random. The bidding function is:

$$b_1 = \frac{c_1 + (c_1^2 + 6)^{0.5}}{3 + R} \qquad \text{if} \qquad 0 < c_i < 1$$

$$b_1 = \frac{2 + 2c_1}{3 + R} \qquad \text{if} \qquad 1 < c_i < 2$$

Where: R -- is a uniform random number generated by Excel's built-in random number generator.

This scenario assumed that both of the Bidders sometimes reduce their equilibrium bid to win the contract. The bidders sacrificed average profit per contract to increase the probability of winning. Simulation results are shown in Figure 16.

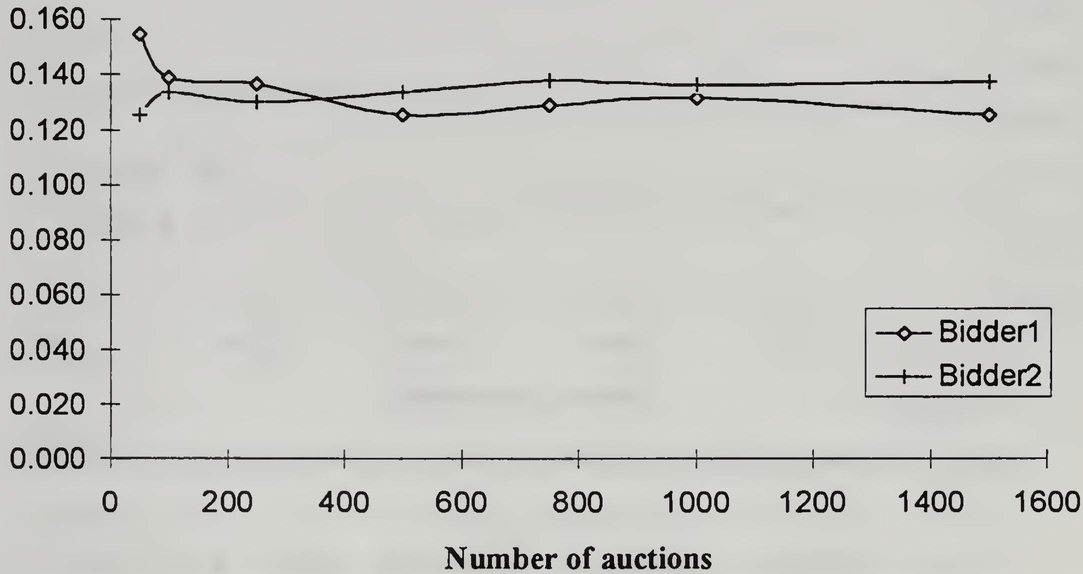

Figure 16 Average profit from bidding -- Bidders used non-equilibrium strategy

This simulation shows that the average profit curves do not follow any defined pattern. Though the average profit curves intersect, the non-equilibrium strategy

does not provide a defined average cost pattern. Figure 16 also demonstrates that both of the bidders had lower payoffs in this case compared to the two previous simulations.

b. Summary

After analyzing the results of the two-bidder computer simulation, it can be observed that:

1. The change in the production cost probability distribution does not substantially change the general results of the FPSBA. However, it does affect the specific values of the bids and the bidders' expected average profit. To better approximate the difference between the expected average profits, a two bidders simulation with a uniform distribution over the range [0, 2] was conducted. The expected average payoff of the bidders was 0.334, which is 80% higher than 0.196 -- the expected average profit with a triangular distribution over the same range.
2. The equilibrium strategy maximizes the bidders' expected average profit; deviations from the equilibrium strategy cause an expected loss for the deviant bidder.
3. If the deviant bid is below the equilibrium strategy, then both of the bidders had lower expected payoffs. If the deviant bid is above the equilibrium strategy then expected profit is higher for the equilibrium strategy bidder.

4. Experimental Setting FPSBA with Three Bidders

This simulation was conducted to find the effect of changing the number of bidders on the game equilibrium. The simulation assumes that three participants bid for a contract; the lowest bid wins. The bidders' production costs are assumed to be distributed according to a triangular distribution between [0, 2], with mode value [1].

During the simulation, three different scenarios were analyzed as in two bidder simulation. In the first scenario, all the bidders used the equilibrium strategy. In the second scenario, two bidders used the equilibrium strategy while one bidder used a non-equilibrium strategy. In the third scenario, all the bidders used a non-equilibrium strategy.

The scenarios were designed to support the theoretical finding that the bidders' equilibrium strategy is dominant. In other words, the equilibrium strategy maximizes the payoff to all the bidders. Deviating from the equilibrium strategy reduces the payoff for the deviating bidder. The numerical simulation results are shown in Appendix C.

a. Graphic Presentation of the Computer Simulation Results

The simulation was used to find an equilibrium in the bidding for contract game. The bidders' average expected profit demonstrates the presence of an equilibrium strategy. If the bidders achieve equal average profit in a series of auctions conducted under the same conditions, this outcome represents the equilibrium of the game.

Figure 17 shows that there exits a well-defined equilibrium in these games. The bidders' average profit approaches 0.105 as the number of simulated auctions approaches 1500.

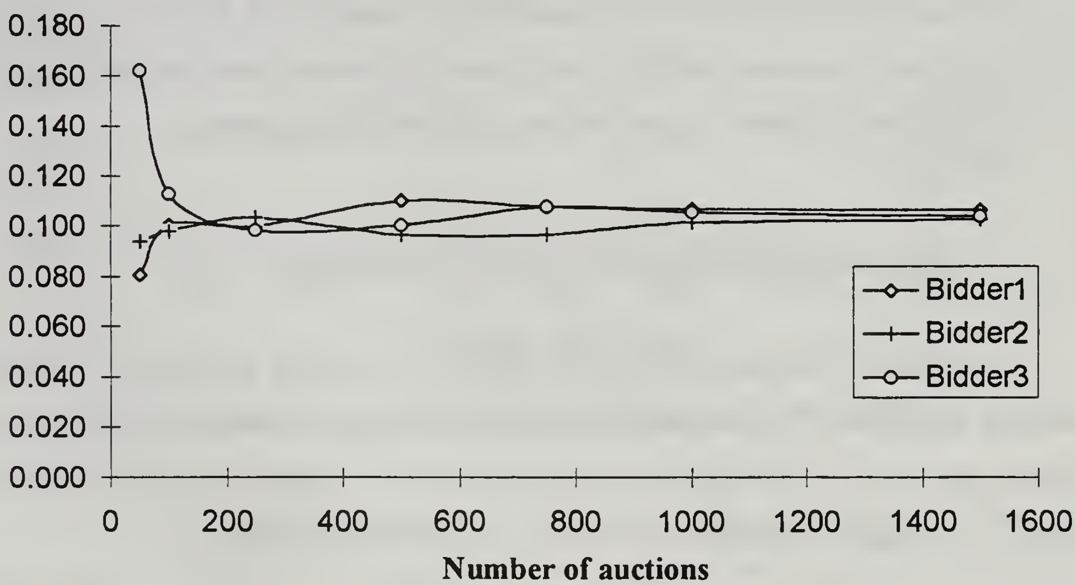

Figure 17 Average profit from bidding -- Bidders using equilibrium strategy

In next simulation, one of the bidders -- Bidder 3 -- used a non-equilibrium strategy. This simulation assumed that Bidder 3's bid was 0.1 more than the equilibrium strategy.

Bidder 3's bidding function is given by:

$$b_i = \frac{2c_i + (4c_i^2 + 10)^{0.5}}{5} + 0.1 \qquad \text{if} \qquad 0 < c_i < 1$$

$$b_i = \frac{2 + 4c_i}{5} + 0.1 \qquad \text{if} \qquad 1 < c_i < 2$$

The simulation revealed that the deviant bidder's profit was lower compared to the equilibrium strategy. The bidders playing the equilibrium strategy -- Bidders 1 and 2 -- received higher profits in this game than in the game where all players used the equilibrium strategy.

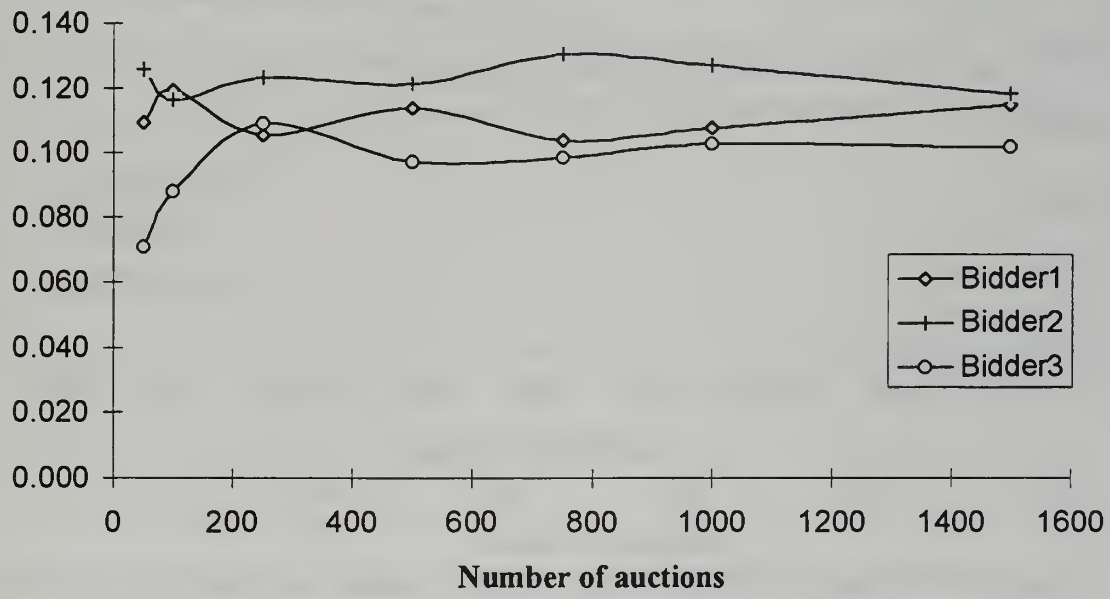

Figure 18 Average profit from bidding -- Bidder 3 using a non-equilibrium strategy

To verify this result, another simulation was conducted with a different non-equilibrium strategy. In this case Bidder 3 used a non-equilibrium strategy described with the bidding function:

$$b_i = \frac{2c_i + (4c_i^2 + 10)^{0.5}}{5} - 0.1 \qquad \text{if} \qquad 0 < c_i < 1$$

$$b_i = \frac{2 + 4c_i}{5} - 0.1 \qquad \text{if} \qquad 1 < c_i < 2$$

This simulation revealed that the deviant bidder's profit was lower compared to the equilibrium strategy. However, in this situation the competing bidders also had lower profits. The deviant 'low" bids helped Bidder 3 win more contracts than before. This reduced the profit for all the bidders, including Bidder 1 and 2 who played the equilibrium strategy.

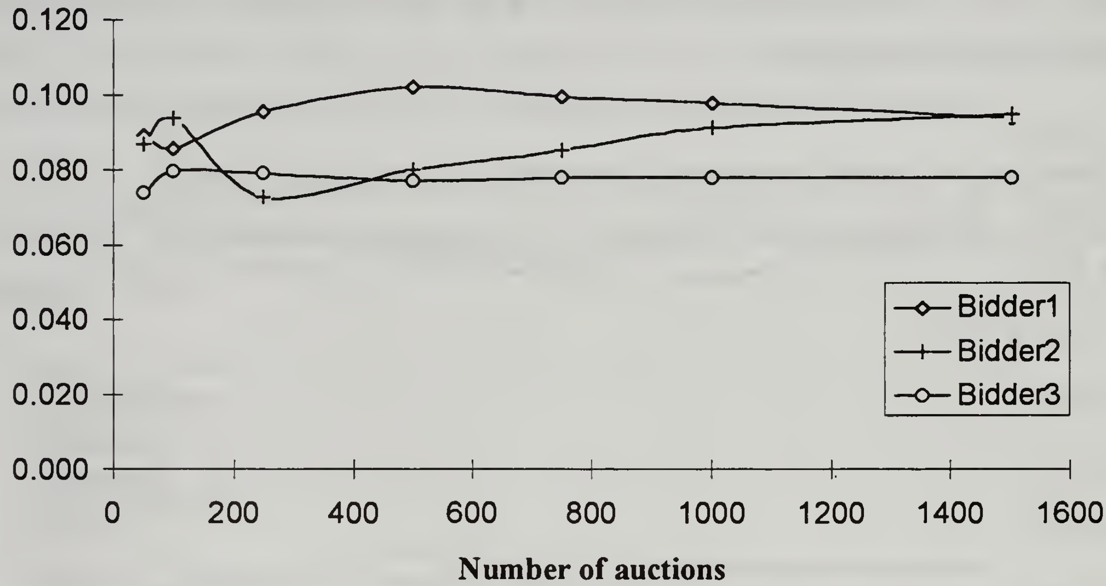

Figure 19 Average profit from bidding -- Bidder 3 using a non-equilibrium strategy

To further verify the existence of an equilibrium strategy, the next scenario assumes all bidders deviate from their equilibrium strategy. The simulation was used to ascertain if any bidder could achieve a higher average payoff in this case.

This simulation assumed a random deviation from equilibrium, as opposed to a systematic non-equilibrium strategy. The bidding functions are given by:

$$b_i = \frac{2c_i + (4*c_i^2 + 10)^{0.5}}{5 + 0.5*\text{Rand()}} \qquad \text{if} \qquad 0 < c_i < 1$$

$$b_i = \frac{2 + 4*c_i}{5 + 0.5*\text{Rand()}} \qquad \text{if} \qquad 1 < c_i < 2$$

Figure 20 shows that the average profit curves in this case follow a defined pattern. The random but symmetric bidding strategy generates this outcome. If the bidders used non-symmetric random deviations from their equilibrium strategy, then the expected average profit curve would not follows any particular pattern.

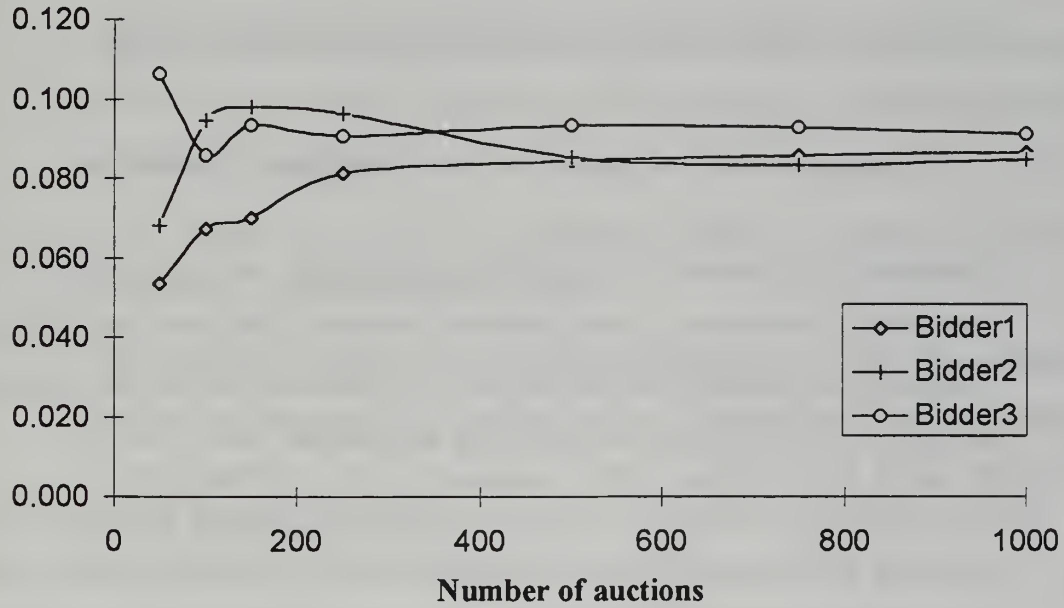

Figure 20 Average profit from bidding -- All Bidders using a non-equilibrium strategy

Figure 20 also demonstrates that all bidders had lower payoffs in this case, compared to the equilibrium strategy simulation.

b. Summary

After analyzing the results of the three-bidder computer simulation, it can be claimed that:

1. The simulation converged to an average payoff of 0.105 after 1500 simulated auctions. This result confirms the theoretical prediction that number of bidders is one of the decisive elements. The higher the number of the bidders, the lower their expected profit.
2. The equilibrium strategy maximizes the bidders' expected average profit; deviations from the equilibrium strategy reduce profit for the deviant bidder, as in the two bidder simulation.
3. If the deviant bid is below the equilibrium strategy, then all bidders had lower expected payoffs. If the deviant bid is above the equilibrium strategy, then the equilibrium strategy bidders have a higher expected profit on average.

5. Conclusions from the Simulations with Triangular Distribution

1. Changing the production cost distribution did not affect the general results, but substantially altered the expected numerical outcomes of the simulated FPSBA, both in two and three bidder simulations. The difference between the expected average profit derives from the probability distributions' characteristics.
2. The equilibrium strategy maximizes the bidders' expected average profit; deviating from the equilibrium bid reduces profit for the deviant bidder. The non-equilibrium strategy reduces the average profit for all bidders if the deviant bid is below the equilibrium bid.
3. Adding additional competitors to the game does not change the nature of the game; the additional competitor reduces both the average winning bid and the average profit.

D. CONCLUSIONS

The FPSBA simulation revealed the bidders' fundamental characteristics in the bidding for contract games. The model assumptions helped to isolate the most influential factors, however assuming risk neutrality for the bidders is dubious. The computer simulation results affirm:

1. The equilibrium strategy maximizes the bidders expected average profit; it appear to be a dominant strategy. Changes in the cost distribution do not effect this pattern; however, the specific values of the expected profit did change with the cost distribution.

2. The number of computing bidders is another influential factor in FPSBA. The additional competitors reduce both the average bid and the average profit. However, the additional competitor did not change the nature of the game.

3. Deviations from the equilibrium strategy cause an expected loss for the deviant bidder. If the deviation is below the equilibrium strategy, all the bidders receive lower expected payoffs.

4. If the deviant bid is above the equilibrium strategy, it increases the expected profit of the equilibrium strategy bidder.

5. The bidding for contract game simulation appears to be valid. The difference between the computed and simulated results is insignificant. The computations in Appendix B and the simulation results will converge if the number of simulated auctions increases.

IV. CONCLUDING OBSERVATIONS

To motivate the conclusions with recommendations, the research questions posed in Chapter I will be reviewed and discussed. In addressing each of the primary and subsidiary research questions, this Chapter will discuss the theoretical basis of the FPSBA simulation described in Chapter II. The experiment's findings, presented in Chapter III, will be the conclusions' foundation. Following these discussions, recommendations will be provided regarding the application of the FPSBA simulation in the HDF's acquisition practice; and further research areas will also be outlined.

A. FPSBA RESEARCH QUESTIONS

As presented in Chapter I the primary research question was: *How do profit maximizing suppliers choose their bids in a competitive environment?* Before answering this question each subsidiary question will be answered.

1. Subsidiary Questions and Discussion

This discussion will clarify the answer to the primary question and provide the foundation for recommendations.

a. Subsidiary Question 1

Does the FPSBA have a game equilibrium, and if it has, what are the equilibrium strategies of the bidders? Chapter II of the thesis analyzed and reviewed the auctioning theory. The bidding for contract game had been classified as a specific instance of a static non-cooperative game with incomplete information. The theory predicts that these games have an equilibrium. This Thesis found and analyzed the symmetric linear equilibrium in bidding for contract games.

Chapter II derived the equilibrium strategies for the bidder when the bidders' production cost has either a uniform or triangular distribution. The resulting

equations -- called bidding functions -- were used to conduct the FPSBA simulations. Several assumptions were made about the bidders' information space and strategic behavior. These assumptions provide a better understanding of the game and made it easier to construct the bidding for contract game model. The developed model was used to compose the simulation algorithm and the code for the computer simulations. This thesis used Excel 5.0 spreadsheet software and Visual Basic language to conduct the simulations.

The first simulation was conducted with a uniform cost distribution. The first of the three simulated FPSBA scenarios defined the existence of the theoretically predicted game equilibrium. The simulation conducted with the equilibrium bidding function supported the theoretical prediction. The simulated auctions provided approximately equal average expected payoffs for the bidders. The two-bidders' equilibrium strategy simulation result was used to validate the applied simulation model. The expected difference between the two bids, computed by using order statistics, is the expected average profit for the winner. The order statistics results were virtually identical with the simulation's results. Considering both the theoretical predictions, and the simulations conducted both for the uniform and triangular distributions, there exists a well-defined symmetric Bayesian Nash equilibrium in the FPSBA games.

The revealed game equilibrium and the proven existence of the equilibrium strategy provide the opportunity to predict some behavior of the potential suppliers. However, the user should be aware of the probabilistic character of the simulated game. It cannot forecast the actual behavior of the bidders in a particular auction but it can approximate the general pattern and the expected average wining bid with a reasonable accuracy.

b. *Subsidiary Question 2*

Do the bidders have a dominant strategy in First Price Sealed Bid Auctions? An equilibrium strategy should also simultaneously maximize the bidders' profits. If a strategy maximizes the bidders' payoff, no-bidder is willing to deviate from the

strategy. This is an equilibrium of the game. This equilibrium condition was explored in simulations where one of the bidders deviated from the equilibrium strategy. Another scenario explored the consequences if all bidders used non equilibrium strategies.

The simulation results supported the theoretical prediction. Deviations from the equilibrium strategy caused an expected loss for the deviant bidders. If the deviation in bidding was negative i.e., bids were below the equilibrium strategy bids, then the bidders achieved a lower expected payoff. If just one bidder deviated from the equilibrium strategy in a positive direction, than the equilibrium strategy bidder's expected profit was higher on average.

These findings support the assumption about the existence of an equilibrium strategy in FPSBA. However, this equilibrium strategy does not satisfy the strong criteria of equilibrium; the bidder should have a best bid regardless of how high he believes the others will bid. 'Each bidder in the FPSBA chooses his best bid given his guess (correct in equilibrium) of the decision rule being followed by the other bidders." [Ref. 4] That is why, the bidders in a FPSBA do not have a dominant strategy.

The game equilibrium satisfies the weak criterion of Nash equilibrium. This Thesis did not analyze the entire scale of the potential equilibrium opportunity. The assumed information symmetry and preference equality made it possible to analyze only the symmetric Nash equilibrium in the game.

c. *Subsidiary Question 3*

How does the change in bidders' numbers affect the outcome of the FPSBA? One of the most influential factors affecting the game outcome is the number of bidders involved in the FPSBA. The theory predicts this both for the uniform and triangular cost distribution. The bidders' number is always present in the general form of the bidding functions that supports the predictions. The conducted simulations supported the theoretical prediction.

To extend the simulations' finding, the bidding function derived for n bidders was used to represent how changing the participants' numbers affects the auctions'

outcome. Figure 21 shows the changes in bids as a function of the bidders involved in the game. The graph has been constructed with constant production costs, for the uniform distribution over the cost range [0, 1].

Figure 21 was created for two costs, $c_1 = 0.8$ and $c_2 = 0.4$. As the Figure indicates, the bidders' expected bids decrease if the bidders' number increases from two and to ten. Adding bidders beyond ten will not significantly decrease the expected bid.

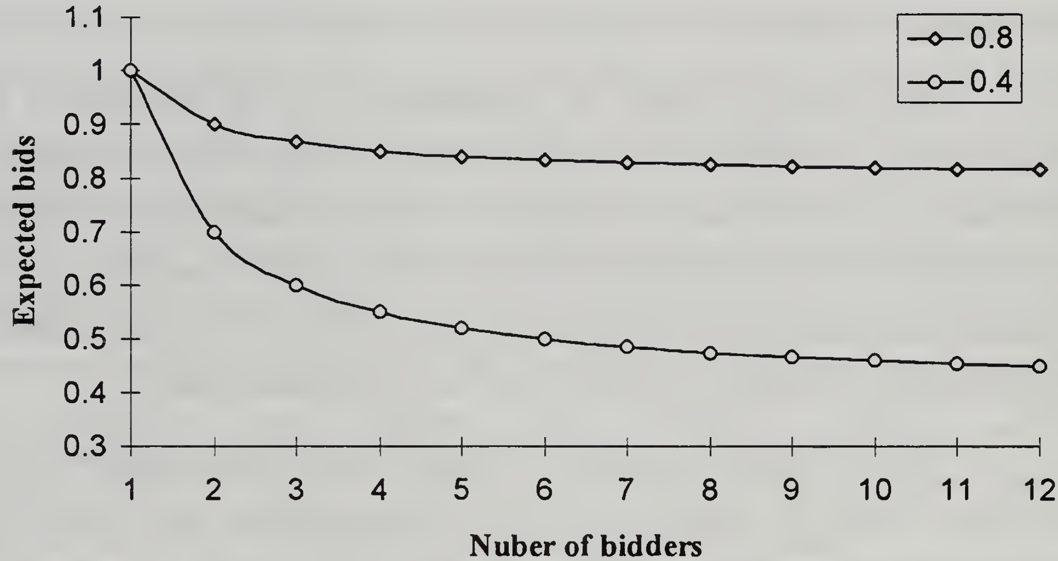

Figure 21 Expected bid as function of the bidders' number

The triangular distribution may better approximate the real costs than the uniform distribution. Figure 22 shows the changes in bids as function of the bidders involved into the game. This graph has been created with constant production cost, for the triangular distribution over the cost range [0, 2].

Figure 22 was completed for two constant cost cases, $c_1 = 0.8$ and $c_2 = 1.4$. As the Figure shows, the bidders' expected bid decreases dramatically as the bidders' number increases from two to ten. Adding bidders beyond ten will not significantly decrease the expected bid.

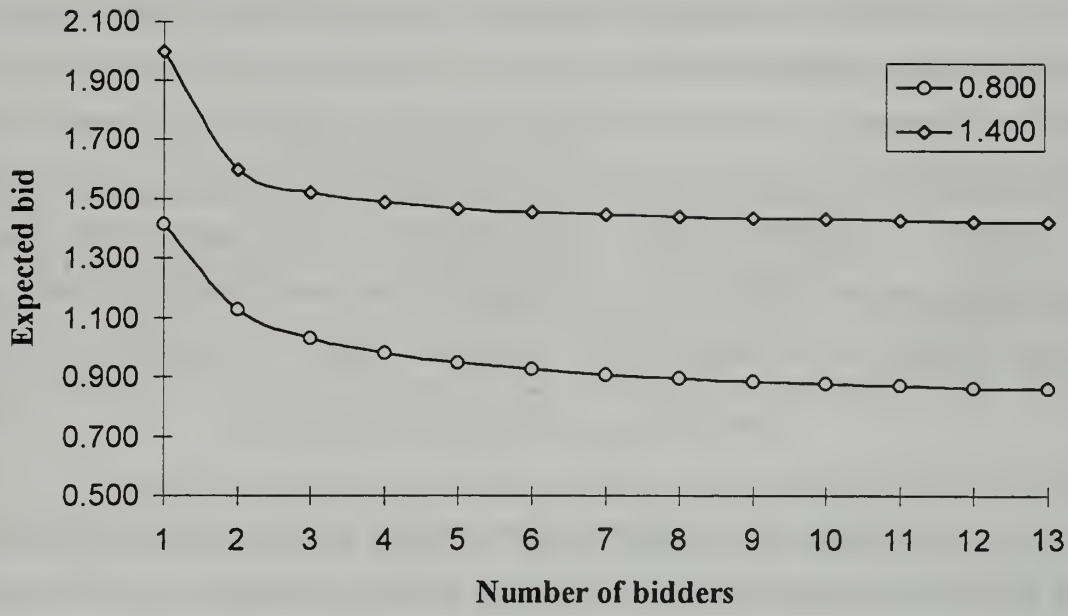

Figure 22 Expected bid as function of the bidders' number

The participants' number influences the bidders' decision in FPSBA. The bidders consider it a potential factor affecting the probability of winning the auction. The higher the number of bids, the higher the probability that any given bid will not be the lowest bid. The expected bid decreases as the participants' number increases.

d. Subsidiary question 4

How does changing the cost distribution affect the outcome of the FPSBA? Changing the probability distribution of production cost did not substantially change the pattern of the FPSBA. The change, however, had a significant effect on the value of the bids and the bidders' expected average profit. To define the difference between the expected average profits, a simulation was conducted for uniform cost distribution of [0, 2] and two bidders. The bidders' expected average payoff was 0.668, which is 80% higher than the expected average profit with a triangular distribution over the same range with mode [1].

The following table presents how the change in bidders' numbers effects the outcome of the simulated FPSBA.

	Expected average payoff		
Equilibrium strategy games	Uniform cost distribution over the range [0, 1]	Uniform cost distribution over the range [0, 2]	Triangular cost distribution over the range [0, 2] with mode [1]
Two Bidders	0.168	0.334	0.196
Three Bidders	0.083	0.167	0.105

Table 1 Changes in expected average profit

The change in the cost distribution assumption has a substantial influence on the FPSBA outcomes. However, it is not just the cost distribution that affects the bidding outcomes. The bidders' perception about the cost for the other participants influences their bids as well. This Thesis assumed that the cost distributions are common knowledge among the bidders; it did not analyze the potential influence if the bidders have different cost assumptions.

In reality, bidders and the buyer have asymmetric information about the cost distribution. Each bidder has private information about their cost, but this cost is a random variable influenced by several factors. The suppliers send signals about their cost to the buyer and to each other. It is in the senders' interest to distort this information, mostly in an increasing direction. The bidders and the buyer have to be aware of this strategic misrepresentation and strategic misinterpretation.

e. Subsidiary Question 5

How can HDF use the findings of this Thesis in their acquisition practice? Discussing subsidiary question 5, this section will make recommendations for the HDF's procurement practice. Some finding of this Thesis can be applied without any further considerations. However, several findings could be used as guidelines; these finding should refer to the actual situation.

Both the theory and the simulation revealed that the equilibrium strategy is the strategy that maximizes the bidders' expected profit. The suppliers will likely use this strategy when they formulate their bids in a real bidding for contract game. The procurement practice can use this finding by projecting the expected winning bid. The equations can use these projections to forecast the expected spending on procurement, and to prepare negotiators for contract negotiations. The essence of these projections is forecasting the accurate expected cost range. The simulation cannot correct forecasts errors.

Findings regarding the number of bidders and their effect on the expected bid can be used to establish the number of invited bidders. This thesis assumed that the HDF does not incur costs during the tender evaluation process because reliable cost data was missing. However, tender evaluation is not a cost-free procedure. Having data about the cost of evaluation, the HDF can conduct a cost and benefit analyses based on these data. Comparing the marginal costs and benefits the HDF may decide the number invited to tender.

2. Primary Question and Discussion

Bidders in a FPSBA offer their bids based on the information they have available. This Thesis assumed symmetry of information among the bidders. It further assumed that the most important information -- the bidder's own production cost -- is private information. The two most influential factors have been considered in building the FPSBA model. The number of bidders invited has been explicitly included in the model. The bidders' production cost assumption, the second influential factor, has been implicitly included in the model by assuming the production cost distribution.

Preparing their bids, FPSBA bidders consider their own production cost, the other bidders' production cost distribution, and the number of competing bidders. They predict their own future production cost. They project their competitors' production costs based

on experience. The bidders know the potential competitors' bidding habits and production potentials.

This Thesis approximated the bidders' mutual experience and knowledge by a probability distribution over production cost. The probability distribution of cost was either uniform or triangular. The uniform distribution of production cost is applicable when the buyer has no ex-ante information about the competing bidders' cost. The triangular distribution applies when the buyer has some information about the bidders' cost.

The assumed profit maximizing behavior made it possible to derive the equilibrium bidding functions. These functions describe the dilemma bidders face in preparing their bid. The bidders have to bid to maximize their profit, but the bid should be low enough to have a reasonable chance to win the auction. The bidders have to tradeoff between the probability of winning with the expected profit if they win the FPSBA.

B. AREAS FOR FURTHER RESEARCH

There are several areas of research that this Thesis did not explore. They are as follows:

1. The Thesis assumed that the bidders are risk neutral. Altering this assumption to allow bidders to be risk averse would illustrate another aspect of the bidders' behavior in FPSBA.
2. The FPSBA practice uses several methods to influence the bidders' behavior. Preferred measures are the reservation price and price discrimination among the bidders. Further research would highlight the effect of these measures on the FPSBA outcomes.
3. The FPSBA was assumed to be a static game. The bidders send signals to each other and the buyer modifies its behavior after receiving and decoding these

signals. Further study could analyze the FPSBA activity as a dynamic Bayesian game and develop further understanding of these games.

C. FINAL THOUGHTS

The FPSBA is an ancient market institution which has been used for thousand of years. The government contracting practice uses FPSBA because it furnishes an efficient solution to the economic and resource allocation problems. The bidders' behavior in FPSBA is determined by several factors and perceptions. Using the game theory approach in microeconomics, this behavior can be analyzed and described. The game simulation is one method to explore this issue. However, one should not confuse the simulation with reality. A stochastic system model provides data which one can expect on average. These models do not prescribe the players' behavior; they forecast and describe average behavior over time.

APPENDIX A. THE BIDDING FUNCTIONS FOR N BIDDERS

This Appendix derives the general form bidding function for the uniform and triangular cost distributions.

UNIFORM COST DISTRIBUTION

Suppose Players 1, 2, 3, ..., n adopt the strategy $b(\cdot)$, and assume that $b(\cdot)$ is strictly increasing and differentiable. Then for a given value of c_i, player i's optimal bidding strategy solves:

$$\max[(b_i - c_i)* \text{Prob}(b_i < b(c_1), \ldots, b_i < b(c_n))]$$

n - denotes the number of bidders.

Using the same approach as in case of three bidders, we can define the probability:

$$\text{Prob}(b_i < b(c_{i+1}), \ldots, b_i < b(c_n)) = (1 - b^{-1}(b_i))^{n-1}$$

The first order condition for player i's optimization problem is therefor:

$$d[(b_i - c_i)* (1 - b^{-1}(b_i))^{n-1}]/dbi = 0$$

$$(1 - b^{-1}(b_i))^{n-1} + (b_i - c_i)*(n - 1)*(1 - b^{-1}(b_i))^{n-2}*d(1 - b^{-1}(b_i))/db_i = 0$$

Implying the same assumption as in case of three bidders, we substitute $b_i = b(c_i)$ into the first order condition, yielding:

$$(1 - b^{-1}(b(ci))^{n-1} + (b(ci) - c_i)*(n - 1)*[1 - b^{-1}(b(ci))]^{n-2}*d(1 - b^{-1}(b(ci))/db_i = 0$$

where: $b^{-1}(b(ci)) = c_i$ and $d(1 - b^{-1}(b(ci))/db_i = - 1/b'(c_i)$

Thus, $b(\cdot)$ must satisfy the first order differential equation:

$$(1 - c_i)^{n-1} - (b(ci) - c_i)*(n-1)* (1 - c_i)^{n-2}*1/b'(c_i) = 0$$

$$(1 - c_i)^{n-1} = (b(ci) - c_i)*(n-1)* (1 - c_i)^{n-2}*1/b'(c_i)$$

We can express this equation as:

$$b'(c_i)*(1 - c_i)^{n-1} - (n - 1)* b(ci) = - (n - 1)*c_i$$

The left hand side of this equation can be rewritten as:

$$b'(c_i)*(1 - c_i)^{n-1} - (n - 1)*b(ci) = 1/(1 - c_i)^{n-2}*d(b(c_i)*(1 - c_i)^{n-1})/dc_i$$

So, the original equation can be rewritten as:

$$1/(1 - c_i)^{n-2} * d(b(c_i)*(1 - c_i)^{n-1})/dc_i = - (n - 1)*c_i$$

$$d [(b(c_i)*(1 - c_i)^{n-1}]/dc_i = -c_i*(n - 1)*(1 - c_i)^{n-2}$$

Integrating both sides of this differential equation, the right hand side by parts yields:

$$\int d [(b(c_i)*(1 - c_i)^{n-1}]dc_i = \int c_i* (- (n - 1)*(1 - c_i)^{n-2})dc_i$$

$$b(c_i)*(1 - c_i)^{n-1} = c_i*(1 - c_i)^{(n-1)} - \int(1 - c_i)^{n-1}\, dc_i$$

$$b(c_i)*(1 - c_i)^{n-1} = c_i*(1 - c_i)^{n-1} + [(1 - c_i)^n]/n + k$$

To determine k, we have to use the boundary conditions. That is, $b(c_i) >= c_i$. If $c_i = 1$, b(1) is finite, which is true. Thus k = 0

Substituting this value of k into the original equation, we find the bidding function for i.

$$b(c_i) = c_i + (1 - c_i)/n = (1 + (n -1)*c_i)/n$$

Under the assumption that the players' strategies are strictly increasing and differentiable, we have a linear and symmetric Nash equilibrium in the n person bidding game.

Applying the same method, the bidding function for the interval [0, 1] can be derived for the cost range [h, l]. In this case, the bidding function takes the form

$$b(c_i) = ((h - l) + (n - 1)*c_i)/n$$

Where: l < h and
l - the lower limit of the distribution
h - the upper limit of the distribution

COST DISTRIBUTION IS TRIANGULAR

In general, a random variable X has a triangular distribution if its probability density function f(x) is given by

$$f(x) = \begin{cases} 2*(x - l)/(h - l)*(m - l) & l < x < m \\ 2*(n - x)/(h - l)*(h - m) & m < x < h \\ 0 & \text{elsewhere} \end{cases}$$

Where: $l < m < h$ and

- l - the lower limit of the distribution
- h - the upper limit of the distribution
- m - the mode of the distribution.

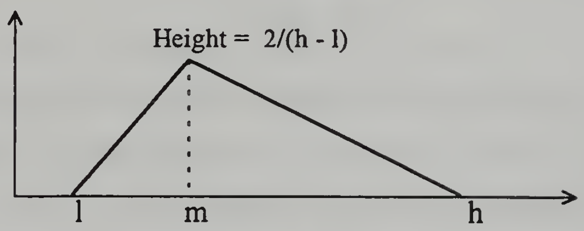

Figure 22 PDF of the triangular distribution

The cumulative distribution function F(x) of the triangular distribution is given by

$$F(x) = \begin{cases} 0 & x < l \\ (x - l)^2/(h - l)*(m - l) & l < x < m \\ 1 - (h - x)^2/(h - l)*(h - m) & m < x < h \\ 1 & x > h \end{cases}$$

Player i's optimal bidding strategy solves:

$$\max(b_i - c_i) * \text{Prob}\{b_i < b(c_{i+1}), \ldots, b_i < b(c_n)\}$$

The number of players in the bidding game is n, so the probability that $b_i(c)$ is the lowest bid is defined by:

$$\text{Prob}\{b_i < b(c_{i+1}), \ldots, b_i < b(c_n)\} = (1 - b^{-1}(b_i))^{n-1}$$

The first order condition for player i's optimization problem is therefor:

$$d[(b_i - c_i)*(1 - b^{-1}(b_i))^{n-1}]/dbi = 0$$

The triangular distribution has special characteristics. It has two different distributions over the interval [l, h]. The dividing limit of the interval is the mode of the distribution [m]. This Appendix will define the bidding function for bidders if their cost falls in different intervals over [l, h].

Definition of the bidding function for cost interval [l, m]

The bidders' expected profit $E(\pi)$, has a maximum if the $dE(\pi)/db = 0$

$$E(\pi) = (b - c)*(1 - F(x))^{n-1}$$

$$d[(b - c)*(1 - (b - l)^2/(h - l)*(m - l))^{n-1}]/db =$$

$$= [K1 - (b - l)^2]^{n-1} + (b - c)*(n - 1)\ [K1 - (b - l)^2]^{n-2}*(-2*(b - l)) = 0$$

Where: $K1 = (h-l)*(m-l)$

Factoring out $[K1 - (b - l)^2]^{n-2}$ yields:

$$K1 - (b - l)^2 -2*(n -1)\ (b - c)*(b - l) = 0$$

$$K1 - (b^2 - 2b*l + l^2) - 2*(n - 1)(b^2 - c*b - b*l + c*l) = 0$$

$$-[2*(n - 1) + 1]*b^2 + 2*[n*l + (n - 1)*c]*b + K1 - l^2 - 2*(n - 1)*c*l = 0$$

$$-[(n - 1) + 0.5]*b^2 + [n*l +(n - 1)*c]*b +[K1 - l^2 - 2*(n - 1)*c*l]/2 = 0$$

Solving for b using the quadratic formula, we get the general formula for the bidding function, when the cost distribution is triangular and actual costs are in the interval **[l, m]**:

$$b = \frac{n*l + (n - 1)*c + \{(n*l + (n - 1)*c)^2+2*[((n - 1) + 0.5)*(K1 - l^2 - 2*(n - 1)*c*l)]\}^{0.5}}{2*[(n - 1) + 0.5]}$$

Using this formula, we can derive all the necessary bidding functions by substituting in the actual parameter values.

Definition of the bidding function for cost interval [m, h]

The bidders' expected profit $E(\pi)$, has a maximum if the $dE(\pi)/db = 0$.

$$E(\pi) = (b - c) * [1 - F(x)]^{n-1}$$

$$d\{(b - c)* \{1- [1 - (h - b)^2/(h - l)*(h - m)]\}^{n-1}/db =$$

$$=1/K2 *\{[(h - b)^2]^{n-1} + (b - c)*(n - 1)\ [(b - l)^2]^{n-2}*-2*(h - b)\} = 0$$

Where: $K2 = (h-l)*(h-m)$

Factoring out $[(h - b)^2]^{n-2}$ yields:

$$(h - b)^2 - [2*(b - c)*(n - 1) *(h - b)] = 0$$

Factoring out (h - b) yields:

$$h - b - 2*(n - 1)*(b - c) = 0$$

Solving for b, gives the general formula for the bidding function when the cost distribution is triangular and costs are in the interval **[m, h]**

$$b = \frac{h + 2*(n - 1)*c}{2*(n - 1) + 1}$$

APPENDIX B. DEFINITION OF EXPECTED COSTS AND BID

This appendix derives the formulas for computing the expected values of the lowest and the second lowest costs, and the lowest bid when the bidders' production costs have uniform distributions.

The game simulation defines the bidders' average expected profit. To validate the simulation result and verify the simulation method, this thesis uses order statistics to determine the expected difference between the lowest and the second lowest cost. The difference is the winners' expected profit.

Let $X_1, \ldots X_n$ be independent identically distributed random variables with PDF f(x) and CDF F(x). Then, the i-th order statistic x_i^* has a PDF [Ref. 13:pg151]:

$$f(x_i^*) = (n!/(i - 1)!*(n - i)!)*(F(x))^{i-1}*(1 - F(x))^{n-i}*f(x)$$

Let $c_1^* = \min(c_1,\ldots,c_n)$ and $c_2^* = \min(c_2,\ldots,c_n)$ be the lowest and the second lowest members of an order statistics $0 < c_1^* < c_2^* < \ldots < c_n^* < 1$. The sample was drawn from a random variable distributed uniformly over the range [0, 1].

The probability density function for the random variable is:

$f(c_i) = 1$ if $0 < x < 1$ and $f(c_i) = 0$ elsewhere.

The cumulative distribution function is:

$$F(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 0 & \text{if } x < 0 \\ 1 & \text{if } x > 1 \end{cases}$$

The expected value of a random variable [Ref.13: pg.: 42]

$$E(x) = \int f(x)*F(x)\,dx$$

Derivation of the expected lowest and the second lowest costs

The PDF of the lowest cost is equal to the PDF of the first order statistic, c_1^*:

$$f(c_1^*) = n!/(n - 1)!*x^0*(1 - x)^{n-1} = n*(1 - x)^{n-1}$$

The expected value of the lowest order statistic $E(c_1*)$ is computed by:

$$E(c_1*) = \int_0^1 x* n*(1 - x)^{n-1}$$

Integrating the expression by parts yields:

$$E(c_1*) = - x*(1 - x)^n \Big|_0^1 + \int_0^1 (1 - x)^n dx = - 1/(n + 1)* (1 - x)^{n+1} \Big|_0^1 = 1/n+1$$

The PDF of the second lowest cost is equal to PDF of the order statistic c_2*:

$$f(c_2*) = n!/(n - 2)!*x*(1 - x)^{n-2} = n*(n - 1)* x*(1 - x)^{n-2}$$

The expected value of the second lowest order statistics

$$E(c_2*) = n*\int_0^1 x^2* (n - 1)*(1 - x)^{n-2}$$

Integrating the expression by part yields:

$$E(c_2*) = - x^2*(1 - x)^{n-1} \Big|_0^1 + 2*\int_0^1 x*n*(1 - x)^{n-1}dx= 2*\int_0^1 (1 - x)^n dx$$

$$E(c_2*) = -2/n+1*(1 - x)^{n+1} \Big|_0^1 = 2/n + 1$$

Having derived the expected values of the lowest and the second lowest order statistics for a random sample drawn from a distribution for n bidders, we can define the expected values of the bidders' profit in simulated cases.

In a two bidders simulation, the expected value of the lowest cost is:

$E(c_1*) = 1/ n +1 \qquad n = 2 \quad E(c_1*) = 1/3 = 0.3333$

The expected value of the second lowest cost is:

$E(c_2*) = 2/n + 1 \qquad n = 2 \quad E(c_2*) = 2/3 = 0.6666$

In a three bidders simulation the expected value of the lowest cost is:

$E(c_1^*) = 1/ n +1 \qquad n = 3 \quad E(c_1^*) = 1/4 = 0.25$

The expected value of the second lowest cost

$E(c_2^*) = 2/n + 1 \qquad n = 3 \quad E(c_2^*) = 2/3 = 0.5$

Definition of the lowest bid's value

To define the lowest bid's expected value, we have to derive the bids' PDF and the CDF. The bids' distribution depends on the cost distribution from which the bidders' costs are drawn. The correlation between the two distributions is define by the bidding function, which takes the general form:

$$b = (1 + (n - 1)*c)/n$$

When the cost distribution is uniform over the range [0, 1], the bids are distributed uniformly over the range [1/n, 1]. We can get this result by transforming the cost distribution range and applying the bidding function.

The PDF for the bids takes the form:

$f(b) = n/(n - 1) \qquad$ if $1/n < b < 1 \qquad$ and $f(b) = 0$ otherwise

The CDF for the bids takes the form:

$$F(b) = \begin{cases} 0 & \text{if } b < 1/n \\ 1/(n - 1)*(n*x - 1) & \text{if } 1/n < b < 1 \\ 1 & \text{if } b > 1 \end{cases}$$

To derive the lowest bid's expected value, we will need the value:

$$1 - F(b) = 1 - (n/n - 1)*x + 1/n - 1 = n/n - 1*(x - 1)$$

Applying the formula for the PDF of the i-th order statistics, define the PDF for the lowest bid $f(b_1^*)$ as

$f(b_1^*) = n!/(n - 1)!*(n/n - 1)^{n-1}*(1 - x)^{n-1}*n/n - 1 = n*(n/n - 1)^n* (1 - x)^{n-1}$

if $\qquad 1/n < b_1^* < b_2^* < b_i^* \ldots < b_n^* < 1$

The expected value of the lowest bid, $E(b_1^*)$ is

$$E(b_1^*) = (n/n - 1)^n \int_{1/n}^{1} x*n*(1 - x)^{n-1}dx = (n/n - 1)^n[-x(1 - x)^n] \Big|_{1/n}^{1} +(n/n - 1)^n * \int_{1/n}^{1} (1 - c)^n \, dx$$

$$E(b_1^*) = (n/n - 1)^n * 1/n*(n - 1/n)^n + (n/n - 1)^n \text{-} (1 - x)^{n+1}/n + 1 \Big|_{1/n}^{1}$$

$$E(b_1^*) = 1/n + (1/n + 1)*(n - 1)/n = 1/n + (n - 1)/n*(n + 1) = 2/n + 1$$

The lowest bid's expected value is given by:

$$E(b_1^*) = 2/n + 1$$

The lowest bid's expected value is equal to the expected value of the second lowest cost as predicted by the auctioning theory.

$$E(b_1^*) = 2/n + 1 = E(c_2^*) = 2/n + 1$$

Having derived the expected value of the lowest bid for n bidders we can define the lowest bids' expected value in simulated cases.

In the two bidders simulation, the expected value of the lowest bid is given by:

$E(c_1^*) = 2/n +1$ $\quad$ $n = 2$ $\quad$ $E(c_1^*) = 2/3 = 0.6666$

In the three bidders simulation, the expected value of the lowest cost is given by:

$E(c_1^*) = 2/ n +1$ $\quad$ $n = 3$ $\quad$ $E(c_1^*) = 1/2 = 0.5$

TWO-PLAYER BIDDING FOR CONTRACT GAME

1.Uniform cost distribution

Scenario 1. Both bidders use the equilibrium strategy

	Total profit		Won auction		Average Profit	
	Bidder 1	Bidder 2	Bidder 1	Bidder 2	Bidder 1	Bidder 2
50	9.43	7.46	28	22	0.337	0.339
100	19.30	13.40	56	44	0.345	0.305
250	42.61	43.02	124	126	0.344	0.341
500	83.48	86.42	248	252	0.337	0.343
750	128.26	127.24	377	373	0.340	0.341
1000	165.45	172.05	491	509	0.337	0.338
1250	209.23	213.69	620	630	0.337	0.339
1500	250.94	252.70	750	750	0.335	0.337

Table 1 Data to Figure 3

	Average payoff		Total payoff	
Auctions	Bidder 1	Bidder 2	Bidder 1	Bidder 2
50	0.189	0.1492	9.43	7.46
100	0.193	0.1340	19.30	13.40
250	0.170	0.1721	42.61	43.02
500	0.167	0.1728	83.48	86.42
750	0.171	0.1697	128.26	127.24
1000	0.165	0.1720	165.45	172.05
1250	0.167	0.1709	209.23	213.69
1500	0.167	0.1685	250.94	252.70

Table 2 Data to Figure 4

Scenario 2. Bidder 1 used a non-equilibrium strategy while Bidder 2 use an equilibrium strategy

	Average payoff		Total payoff	
Auctions	Bidder1	Bidder2	Bidder1	Bidder2
50	0.16	0.22	8.02	10.81
100	0.16	0.23	15.62	22.57
250	0.15	0.21	36.62	53.46
500	0.14	0.21	69.07	107.21
750	0.14	0.21	103.57	160.54
1000	0.14	0.21	140.13	213.96

Table 3 Data to Figure 5

	Average payoff		Total payoff	
Auctions	Bidder 1	Bidder 2	Bidder 1	Bidder 2
50	0.19	0.11	9.57	5.33
100	0.14	0.12	13.90	12.45
250	0.13	0.13	32.40	32.54
500	0.13	0.13	63.76	65.82
750	0.12	0.14	91.16	101.29
1000	0.12	0.13	122.30	133.08

Table 4 Data to Figure 6

Scenario 3. Both bidders used a non-equilibrium strategy

	Average payoff		Total payoff	
Auctions	Bidder1	Bidder2	Bidder1	Bidder2
50	0.123	0.090	6.15	4.52
100	0.121	0.100	12.10	10.04
250	0.112	0.107	28.08	26.85
500	0.113	0.111	56.47	55.73
750	0.107	0.115	79.95	86.25
1000	0.105	0.113	105.04	112.51

Table 5 Data to Fig. 7

2. Triangular cost distribution

Scenario 1. Both bidders use an equilibrium strategy

	Average Profit		Total Payoff	
Auctions	Bidder1	Bidder2	Bidder1	Bidder2
50	0.184	0.165	9.19	8.24
100	0.193	0.177	19.29	17.70
250	0.217	0.169	54.36	42.25
500	0.202	0.196	101.14	98.24
750	0.197	0.201	147.54	150.93
1000	0.197	0.201	197.09	200.67
1500	0.198	0.194	297.26	290.61

Table 6 Data to Fig. 12

Scenario 2. Bidder 1 used a non-equilibrium strategy while Bidder 2 used equilibrium strategy

	Average Profit		Total Payoff	
Auctions	Bidder1	Bidder2	Bidder1	Bidder2
50	0.270	0.232	13.50	11.61
100	0.213	0.256	21.29	25.60
250	0.172	0.267	43.00	66.70
500	0.191	0.257	95.55	128.65
750	0.190	0.258	142.15	193.39
1000	0.190	0.257	190.23	257.24

Table 7 Data to Fig 13

	Average Profit		Total Payoff	
Auctions	Bidder1	Bidder2	Bidder1	Bidder2
50	0.075	0.128	3.74	6.42
100	0.067	0.142	6.68	14.17
250	0.069	0.129	17.21	32.37
500	0.068	0.124	33.99	61.93
750	0.070	0.119	52.73	89.39
1000	0.071	0.121	71.38	121.36

Table 8 Data to Fig. 14

Scenario 3. Both bidders used a non-equilibrium strategy

	Average Profit		Total Payoff	
Auctions	Bidder1	Bidder2	Bidder1	Bidder2
50	0.155	0.125	7.73	6.26
100	0.139	0.133	13.88	13.35
250	0.137	0.130	34.16	32.49
500	0.125	0.134	62.68	66.80
750	0.129	0.138	96.66	103.37
1000	0.131	0.136	131.39	136.14
1500	0.125	0.137	188.19	206.12

Table 9 Data to Fig 15

THREE PLAYER BIDDING FOR CONTRACT GAMES

1. Uniform cost distribution

Scenario 1. All bidders used an equilibrium strategy

	Average profit			Won Auctions			Total Profit		
Auctions	Bidder 1	Bidder 2	Bidder 3	Bidder 1	Bidder 2	Bidder 3	Bidder 1	Bidder 2	Bidder 3
50	0.24	0.25	0.26	13	19	18	3.15	4.69	4.60
150	0.24	0.25	0.25	53	50	47	12.90	12.52	11.80
250	0.25	0.25	0.25	83	72	95	20.59	18.29	23.57
500	0.25	0.26	0.25	168	159	173	41.60	40.85	43.49
1000	0.25	0.25	0.25	358	323	319	88.80	81.21	79.76
1500	0.25	0.25	0.25	536	486	478	132.99	122.08	119.31
2000	0.25	0.25	0.25	692	663	645	172.43	166.34	161.45
2500	0.25	0.25	0.25	876	821	803	219.59	206.58	200.70

Table 10 Data to Fig. 8

	Average Profit			Total Profit		
Auctions	Bidder 1	Bidder 2	Bidder 3	Bidder 1	Bidder 2	Bidder 3
50	0.063	0.094	0.092	3.15	4.69	4.60
150	0.086	0.083	0.079	12.90	12.52	11.80
250	0.082	0.073	0.094	20.59	18.29	23.57
500	0.083	0.082	0.087	41.60	40.85	43.49
1000	0.089	0.081	0.080	88.80	81.21	79.76
1500	0.089	0.081	0.080	132.99	122.08	119.31
2000	0.086	0.083	0.081	172.43	166.34	161.45
2500	0.088	0.083	0.080	219.59	206.58	200.70

Table 11 Data to Fig. 9

Scenario 2. Bidder 3 uses a non-equilibrium strategy while Bidder 1 and 2 use an equilibrium strategy:

	Average profit			Total profit		
Auctions	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3
100	0.097	0.111	0.047	9.70	11.05	4.66
250	0.109	0.104	0.042	27.30	25.88	10.39
500	0.111	0.105	0.040	55.54	52.28	20.19
750	0.110	0.101	0.047	82.64	76.06	35.54
1000	0.112	0.100	0.044	111.83	100.41	44.45
1500	0.108	0.105	0.048	161.53	157.24	71.64

Table 12 Data to Fig. 10

	Average profit			Total profit		
Auctions	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3
100	0.060	0.062	0.051	5.99	6.22	5.07
250	0.062	0.065	0.043	15.55	16.31	10.82
500	0.062	0.071	0.041	31.10	35.43	20.49
750	0.062	0.071	0.041	46.21	53.31	30.65
1000	0.061	0.068	0.041	61.06	68.03	41.17
1500	0.062	0.064	0.042	92.73	96.03	62.52

Table 13 Data to Fig. 11

Scenario 3. All bidders use a non-equilibrium strategy:

	Average profit			Total profit		
Auctions	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3
100	0.052	0.087	0.061	5.17	8.73	6.10
250	0.066	0.065	0.068	16.51	16.35	16.89
500	0.066	0.069	0.068	33.13	34.35	34.17
750	0.066	0.071	0.063	49.26	53.15	47.29
1000	0.066	0.070	0.065	65.52	69.90	64.72
1500	0.067	0.068	0.066	100.00	101.70	99.62

Table 14 Data to Fig. 12

2. Triangular cost distribution

Scenario 1. All bidders use an equilibrium strategy

	Average profit			Total profit		
Auctions	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3
50	0.121	0.141	0.243	6.06	7.05	12.16
100	0.151	0.147	0.170	15.07	14.73	16.97
250	0.151	0.155	0.148	37.65	38.81	37.00
500	0.165	0.145	0.151	82.63	72.45	75.36
750	0.162	0.145	0.162	121.52	108.75	121.39
1000	0.161	0.152	0.159	160.55	151.95	158.52
1500	0.160	0.154	0.156	240.13	231.55	233.68

Table 15 Data to Fig. 17

Scenario 2. Bidder 3 uses a non-equilibrium strategy while Bidder 1 and 2 use an equilibrium strategy

	Average profit			Total profit		
Auctions	Bidder 1	Bidder 2	Bidder 3	Bidder 1	Bidder 2	Bidder 3
50	0.164	0.189	0.106	8.19	9.43	5.32
100	0.179	0.175	0.132	17.91	17.45	13.19
250	0.158	0.185	0.164	39.55	46.20	40.93
500	0.171	0.182	0.146	85.29	91.00	72.84
750	0.156	0.195	0.148	116.81	146.62	110.95
1000	0.162	0.191	0.154	161.78	190.51	154.22
1500	0.173	0.177	0.153	258.78	266.07	228.94

Table 16 Data to Fig. 18

	Average profit			Total profit		
Auctions	Bidder 1	Bidder 2	Bidder 3	Bidder 1	Bidder 2	Bidder 3
50	0.134	0.130	0.111	6.68	6.50	5.53
100	0.129	0.141	0.119	12.85	14.06	11.95
250	0.143	0.109	0.119	35.75	27.26	29.63
500	0.153	0.120	0.116	76.44	59.88	57.76
750	0.149	0.128	0.117	111.68	95.85	87.77
1000	0.147	0.137	0.117	146.70	136.79	117.04
1500	0.141	0.142	0.117	211.02	213.15	175.08

Table 17 Data to Fig. 19

Scenario 3. All bidders use a non-equilibrium strategy

	Average profit			Total profit		
Auctions	Bidder 1	Bidder 2	Bidder 3	Bidder 1	Bidder 2	Bidder 3
50	0.080	0.102	0.159	8.00	10.21	15.93
100	0.101	0.142	0.129	20.24	28.32	25.78
150	0.105	0.147	0.140	31.48	44.13	42.02
250	0.122	0.145	0.136	61.057	72.312	68.089
500	0.127	0.128	0.140	126.92	128.07	140.05
750	0.129	0.125	0.140	193.54	187.68	209.26
1000	0.130	0.127	0.137	259.96	254.38	273.77
1500	0.128	0.131	0.133	384.00	393.99	398.52

Table 18 Data to Fig. 20

1. Simulation program with uniform cost distribution

```
'' simul2 Macro simulation of three bidders auction
' Macro recorded 4/9/97 by Andras I. Kucsma
'' The program has been prepared for n  bidders. However,
' to extend it for more than three bidders the program
' needs some adjustment. It has to be added
' - new subroutines of cost and bid calculation;
' - the selection of the winner must be corrected too.

Sub simul2()
    Application.ScreenUpdating = False
Definition of the variables
    Dim l, h, n, Num, Cost1, Cost2, Cost3 As Variant
    Dim Bid1, Bid2, Bid3, Payoff1, Payoff2, Payoff3 As Variant
    Dim Rand1, Rand2, Rand3 As Variant
Giving initial values to variables
    Sheets("Sheet1").Select
    h = 1
    Range("b1").Select
    ActiveCell.Value = h
    l = 0
    Range("d1").Select
    ActiveCell.Value = l
    n = 3
    Range("e1").Select
    ActiveCell.Value = n
    Num = 0

Start loop enter the required number of loops after "To"
For Num = 1 To 500
  Randomize 'sets the seed number of random number generation to a new value
    ' Cost generation
    Cost1 = Rnd * (h - l) + 1
```

```
    Range("a3").Select
    ActiveCell.Value = Cost1
    Cost2 = Rnd * (h - l) + l
    Range("b3").Select
    ActiveCell.Value = Cost2
    Cost3 = Rnd * (h - l) + l
    Range("c3").Select
    ActiveCell.Value = Cost3

    ' Computation of bids
  Bid1 = ((h - l) + (n - 1) * Cost1) / n
    Range("d3").Select
    ActiveCell.Value = Bid1

  Bid2 = ((h - l)+ (n - 1) * Cost2) / n
    Range("e3").Select
    ActiveCell.Value = Bid2
   Bid3 = ((h - l) + (n - 1) * Cost3) / n
    Range("f3").Select
    ActiveCell.Value = Bid3
   ' Selecting the winning bid

If Bid1 < Bid2 And Bid1 < Bid3 Then GoTo Row1: Else GoTo Row2:

Row1:
    Payoff1 = Bid1 - Cost1
    Range("g3").Select
    ActiveCell.Value = Payoff1
    GoTo Row6:
Row2:
    If Bid2 < Bid1 And Bid2 < Bid3 Then GoTo Row3: Else GoTo Row4:
Row3:
    Payoff2 = Bid2 - Cost2
    Range("h3").Select
```

```
        ActiveCell.Value = Payoff2
        GoTo Row6
Row4:
If Bid3 < Bid1 And Bid3 < Bid2 Then GoTo Row5:
Row5:
 Payoff3 = Bid3 - Cost3
        Range("i3").Select
        ActiveCell.Value = Payoff3
        GoTo Row6
Row6:
        Range("A4").Select
        Selection.EntireRow.Insert
        Range("A3:j3").Select
        Selection.Copy
        Range("A5").Select
        Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, _
            SkipBlanks:=False, Transpose:=False
        Application.CutCopyMode = False
        Range("A3:i3").Select
        Selection.Clear
Next
Row6:
        For I = 1 To 3  ' Loop 3 times.
        Beep   ' Sound a tone.
        Next
End Sub
```

2. Simulation program with triangular cost distribution

```
' simul2 Macro simulation of two bidders auction
' Distribution is triangular
' Macro recorded 4/9/97 by Andras I. Kucsma
' The program has been prepared for n  bidders. However,
' to extend it for more than two bidders the program
```

```
' needs some adjustment. It has to be added
' - new subroutines of cost and bid calculation;
' - the selection of the winner must be corrected too.
'
Sub simul2()
    Application.ScreenUpdating = False
    Dim h, m, l, n, Num, Rand1, Rand2, Cost1, Cost2 As Variant
    Dim Bid1, Bid1a, Bid1b, Bid1c, Bid2, Bid2a, bid2b  As Variant
    Dim Payoff1, Payoff2 As Variant

    ' Giving initial values to variables
    Sheets("Sheet1").Select
    l = 0 ' lower limit of cost range
    Range("b1").Select
    ActiveCell.Value = l

    m = 1 ' mode of the cost distribution
    Range("d1").Select
    ActiveCell.Value = m

    h = 2 'the higher limit of cost range
    Range("f1").Select
    ActiveCell.Value = h
    Bid1a = 0
    Bid1b = 0
    Bod1c = 0

    n = 2 ' number of bidder

    ' Start loop enter the number of required after "To"

    Num = 0
    For Num = 1 To 250
    '
'Computation of costs
    Randomize ' sets the seed number of the random number generator
    ' Computation of Cost1
   Rand1 = Rnd()
    If Rand1 < ((m - l) / (h - l)) Then
        Cost1 = ((Rand1 * (h - l) * (m - l)) ^ 0.5) + l
        Else
```

```
        Cost1 = h - ((1 - Rand1) * (h - l) * (h - m)) ^ 0.5
    End If

'Cost1 to Sheet1
    Sheets("Sheet1").Select
    Range("a3").Select
    ActiveCell.Value = Cost1

    ' Computation of Cost2
    Rand2 = Rnd()
    If Rand2 < ((m - l) / (h - l)) Then
        Cost2 = ((Rand2 * (h - l) * (m - l)) ^ 0.5) + l
        Else
        Cost2 = h - ((1 - Rand2) * (h - l) * (h - m)) ^ 0.5
    End If
' Cost2 to Sheet1
    Sheets("Sheet1").Select
    Range("b3").Select
    ActiveCell.Value = Cost2

' Computation of bids

    Bid1a = (n - 1) + 0.5

 If Cost1 < m Then

        Bid1b = n * l + (n - 1) * Cost1

        Bid1c = Bid1a * (h - l) * (m - l) - (l ^ 2) - (2 * Cost1 * (n - 1) * l)

        Bid1 = (Bid1b + (Bid1b ^ 2 + 2 * Bid1c) ^ 0.5) / (2 * Bid1a)

        Else

        Bid1 = (h + (2 * (n - 1) * Cost1)) / (2 * Bid1a)

    End If
'bid1 to Sheet1
    Sheets("Sheet1").Select
    Range("c3").Select
    ActiveCell.Value = Bid1
```

```
' Computation of Bid2

    If Cost2 < m Then
        bid2b = n * l + (n - 1) * Cost2

        Bid2c = Bid1a * (h - l) * (m - l) - (l ^ 2) - (2 * Cost2 * (n - 1) * l)

        Bid2 = (bid2b + (bid2b ^ 2 + 2 * Bid2c) ^ 0.5) / (2 * Bid1a)

        Else
        Bid2 = (h + (2 * (n - 1) * Cost2)) / (2 * Bid1a)
    End If

'Bid2 to Sheet1
    Sheets("Sheet1").Select
    Range("d3").Select
    ActiveCell.Value = Bid2



'Defining the winner and its Payoff


    If Bid1 < Bid2 Then GoTo Row1: Else GoTo Row2:




Row1:
    Payoff1 = Bid1 - Cost1
    Range("e3").Select
    ActiveCell.Value = Payoff1
    GoTo Row3:



Row2:
    Payoff2 = Bid2 - Cost2
    Range("f3").Select
    ActiveCell.Value = Payoff2
    GoTo Row3
Row3:
    ' Collecting result of the auction in new row

Range("A4").Select
    Selection.EntireRow.Insert
    Range("A3:g3").Select
```

```
        Selection.Copy
        Range("A5").Select
        Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, _
            SkipBlanks:=False, Transpose:=False
        Application.CutCopyMode = False
        Range("A3:F3").Select
        Selection.Clear

    Next

        'End signal
        For I = 1 To 3  ' Loop 3 times.
        Beep   ' Sound a tone.
        Next

    End Sub
```

APPENDIX E. STATISTICAL ANALYSES OF COST DISTRIBUTION

This thesis used the Excel 5.0 software data analyzer package to inspect the distribution of random variables used in the simulations. Selected data series were analyzed using histograms and cumulative probabilities. The results of the analyses are presented in this Appendix.

Analysis of costs used in simulations with uniform distributions

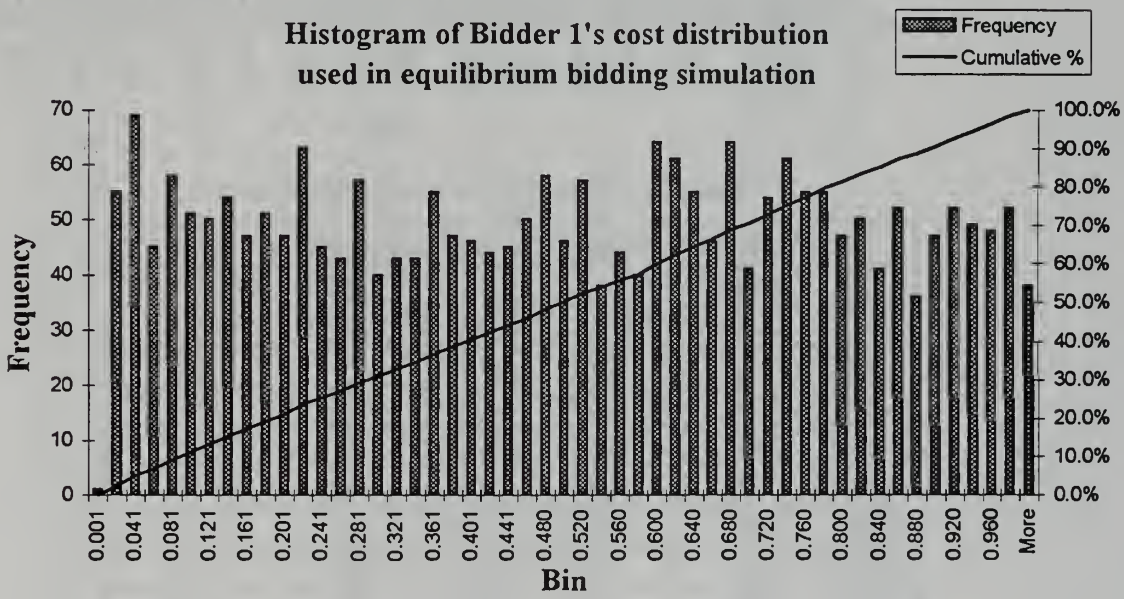

Figure 1 Histogram of uniform distribution

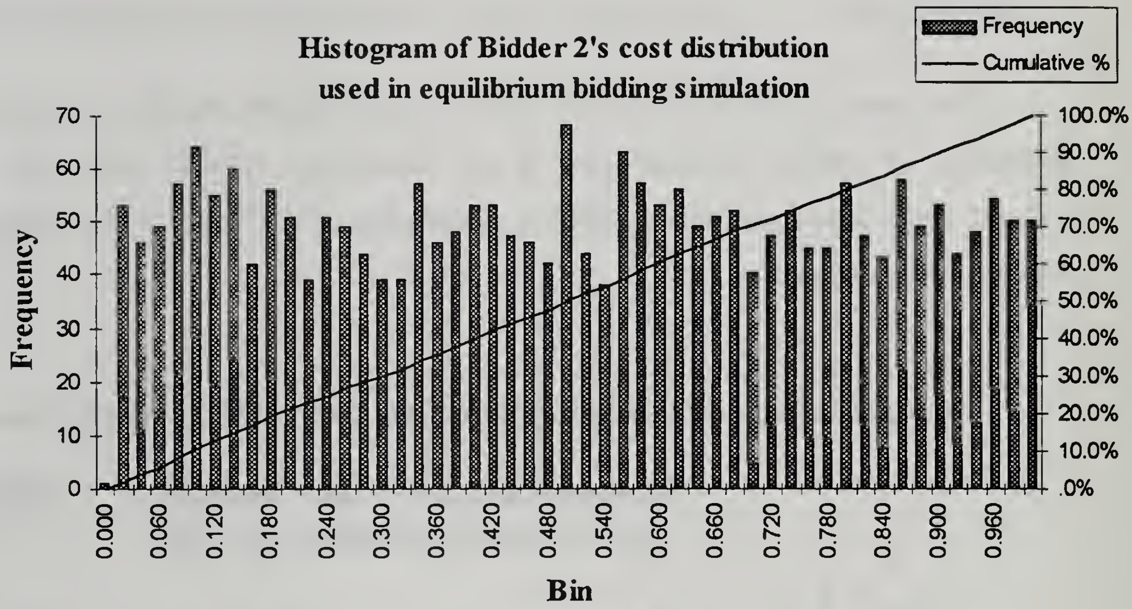

Figure 2 Histogram of uniform distribution

Data of the cost distribution histograms for uniform distribution					
Bidder 1's cost			Bidder 2's cost		
Bin	*Frequency*	*Cumulative %*	*Bin*	*Frequency*	*Cumulative %*
0.001	1	0.0%	0.000	1	.0%
0.021	55	2.2%	0.020	53	2.2%
0.041	69	5.0%	0.040	46	4.0%
0.061	45	6.8%	0.060	49	6.0%
0.081	58	9.1%	0.080	57	8.2%
0.101	51	11.2%	0.100	64	10.8%
0.121	50	13.2%	0.120	55	13.0%
0.141	54	15.3%	0.140	60	15.4%
0.161	47	17.2%	0.160	42	17.1%
0.181	51	19.2%	0.180	56	19.3%
0.201	47	21.1%	0.200	51	21.4%
0.221	63	23.6%	0.220	39	22.9%
0.241	45	25.4%	0.240	51	25.0%
0.261	43	27.2%	0.260	49	26.9%
0.281	57	29.4%	0.280	44	28.7%
0.301	40	31.0%	0.300	39	30.2%
0.321	43	32.8%	0.320	39	31.8%
0.341	43	34.5%	0.340	57	34.1%
0.361	55	36.7%	0.360	46	35.9%
0.381	47	38.6%	0.380	48	37.8%
0.401	46	40.4%	0.400	53	40.0%
0.421	44	42.2%	0.420	53	42.1%
0.441	45	44.0%	0.440	47	44.0%

Bin	*Frequency*	*Cumulative %*	*Bin*	*Frequency*	*Cumulative %*
0.461	50	46.0%	0.460	46	45.8%
0.480	58	48.3%	0.480	42	47.5%
0.500	46	50.1%	0.500	68	50.2%
0.520	57	52.4%	0.520	44	52.0%
0.540	38	53.9%	0.540	38	53.5%
0.560	44	55.7%	0.560	63	56.0%
0.580	40	57.3%	0.580	57	58.3%
0.600	64	59.8%	0.600	53	60.4%
0.620	61	62.3%	0.620	56	62.6%
0.640	55	64.5%	0.640	49	64.6%
0.660	46	66.3%	0.660	51	66.6%
0.680	64	68.9%	0.680	52	68.7%
0.700	41	70.5%	0.700	40	70.3%
0.720	54	72.7%	0.720	47	72.2%
0.740	61	75.1%	0.740	52	74.3%
0.760	55	77.3%	0.760	45	76.1%
0.780	55	79.5%	0.780	45	77.9%
0.800	47	81.4%	0.800	57	80.2%
0.820	50	83.4%	0.820	47	82.0%
0.840	41	85.0%	0.840	43	83.8%
0.860	52	87.1%	0.860	58	86.1%
0.880	36	88.6%	0.880	49	88.0%
0.900	47	90.4%	0.900	53	90.2%
0.920	52	92.5%	0.920	44	91.9%
0.940	49	94.5%	0.940	48	93.8%
0.960	48	96.4%	0.960	54	96.0%
0.980	52	98.5%	0.980	50	98.0%
More	38	100.0%	More	50	100.0%

Table 19 Data to uniform distribution histogram

Analysis of cost used in simulations with triangular distributions

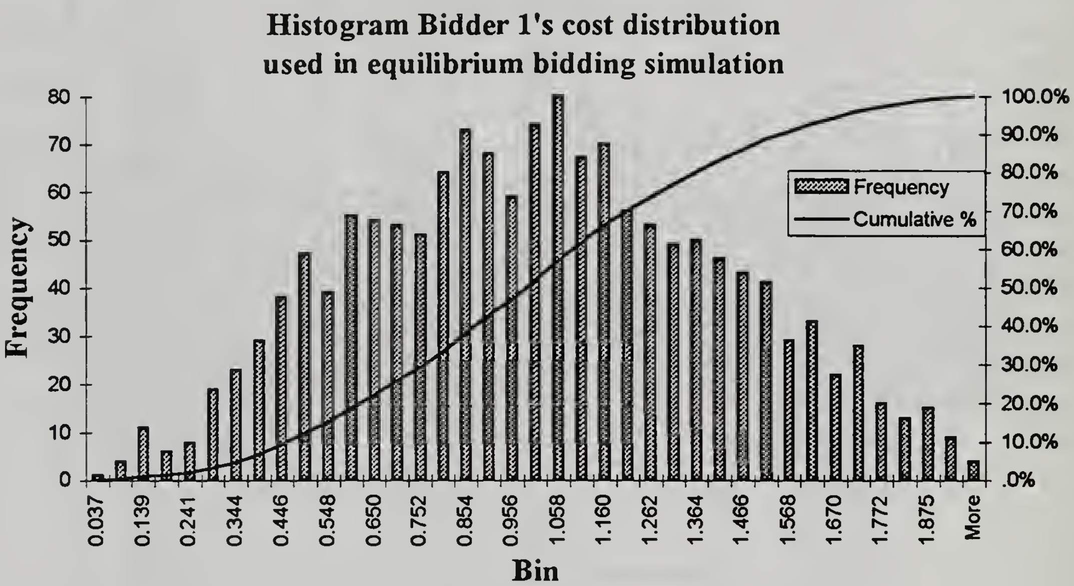

Figure 4 Histogram of triangular distribution

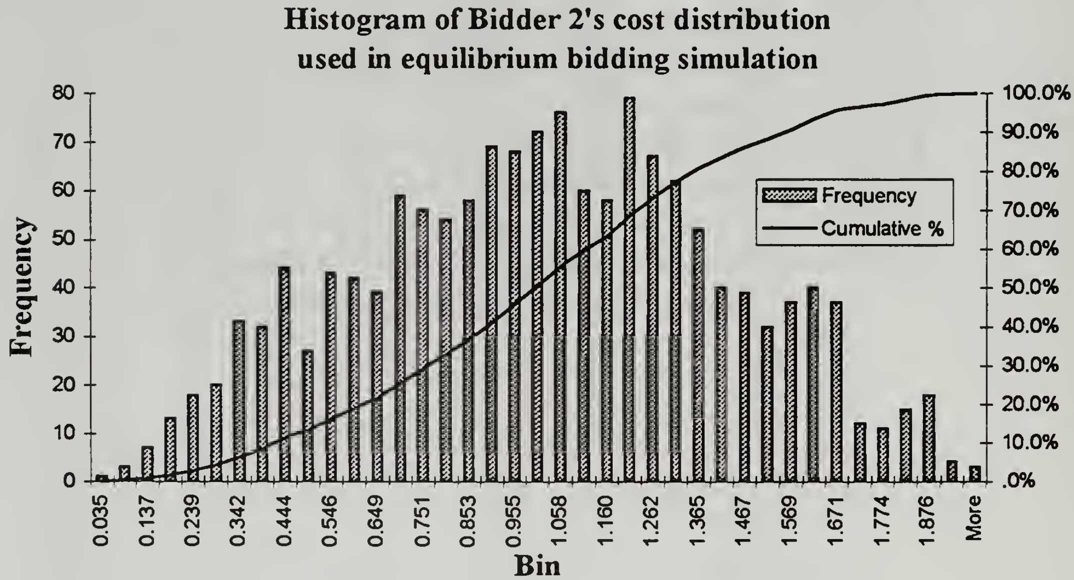

Figure 4 Histogram of triangular distribution

Figures three and four shows that the costs used in the equilibrium game simulation follow the required triangular pattern of distribution. The cumulative percentage represents the cumulative probability distribution of the sample data.

Cost distribution histogram data for triangular distribution					
Bidder 1's cost			Bidder 2's cost		
Bin	*Frequency*	*Cumulative %*	*Bin*	*Frequency*	*Cumulative %*
0.037	1	.1%	0.035	1	.1%
0.088	4	.3%	0.086	3	.3%
0.139	11	1.1%	0.137	7	.7%
0.190	6	1.5%	0.188	13	1.6%
0.241	8	2.0%	0.239	18	2.8%
0.293	19	3.3%	0.291	20	4.1%
0.344	23	4.8%	0.342	33	6.3%
0.395	29	6.7%	0.393	32	8.5%
0.446	38	9.3%	0.444	44	11.4%
0.497	47	12.4%	0.495	27	13.2%
0.548	39	15.0%	0.546	43	16.1%
0.599	55	18.7%	0.597	42	18.9%
0.650	54	22.3%	0.649	39	21.5%
0.701	53	25.8%	0.700	59	25.4%
0.752	51	29.2%	0.751	56	29.1%
0.803	64	33.5%	0.802	54	32.7%
0.854	73	38.3%	0.853	58	36.6%
0.905	68	42.9%	0.904	69	41.2%
0.956	59	46.8%	0.955	68	45.7%
1.007	74	51.7%	1.007	72	50.5%
1.058	80	57.1%	1.058	76	55.6%
1.109	67	61.5%	1.109	60	59.6%
1.160	70	66.2%	1.160	58	63.5%
1.211	56	69.9%	1.211	79	68.7%
1.262	53	73.5%	1.262	67	73.2%
1.313	49	76.7%	1.313	62	77.3%
1.364	50	80.1%	1.365	52	80.8%
1.415	46	83.1%	1.416	40	83.5%
1.466	43	86.0%	1.467	39	86.1%
1.517	41	88.7%	1.518	32	88.2%
1.568	29	90.7%	1.569	37	90.7%
1.619	33	92.9%	1.620	40	93.3%
1.670	22	94.3%	1.671	37	95.8%
1.721	28	96.2%	1.723	12	96.6%
1.772	16	97.3%	1.774	11	97.3%
1.824	13	98.1%	1.825	15	98.3%

Cost distribution histogram data for triangular distribution					
Bidder 1's cost			Bidder 2's cost		
Bin	*Frequency*	*Cumulative %*	*Bin*	*Frequency*	*Cumulative %*
1.875	15	99.1%	1.876	18	99.5%
1.926	9	99.7%	1.927	4	99.8%
More	4	100.0%	More	3	100.0%

APPENDIX F. EXTRACTS FROM THE SIMULATIONS' DATA

This Appendix provides samples from the simulations' data for the first one hundred simulated auctions. The volume of data generated by the simulations precludes presenting the full data series.

1. Two-bidder uniform cost distribution

Scenario 1

#	Cost1	Cost2	Bid1	Bid2	Payoff1	Payoff2
100	0.366	0.842	0.683	0.921	0.3171	
99	0.84	0.375	0.92	0.688		0.3124
98	0.194	0.629	0.597	0.814	0.4029	
97	0.303	0.283	0.652	0.642		0.3584
96	0.817	0.583	0.909	0.792		0.2084
95	0.342	0.811	0.671	0.906	0.3289	
94	0.216	0.948	0.608	0.974	0.3919	
93	0.665	0.633	0.833	0.817		0.1835
92	0.728	0.633	0.864	0.816		0.1836
91	0.334	0.373	0.667	0.687	0.3331	
90	0.055	0.424	0.528	0.712	0.4724	
89	0.555	0.805	0.777	0.902	0.2225	
88	0.177	0.383	0.588	0.691	0.4117	
87	0.102	0.216	0.551	0.608	0.4492	
86	0.559	0.843	0.78	0.921	0.2203	
85	0.956	0.85	0.978	0.925		0.0748
84	0.917	0.46	0.959	0.73		0.2702
83	0.624	0.591	0.812	0.795		0.2046
82	0.979	0.844	0.989	0.922		0.0778
81	0.314	0.209	0.657	0.605		0.3954
80	0.327	0.514	0.663	0.757	0.3367	
79	0.517	0.245	0.759	0.623		0.3773
78	0.188	0.601	0.594	0.8	0.406	
77	0.637	0.286	0.818	0.643		0.3569
76	0.966	0.692	0.983	0.846		0.1539
75	0.322	0.931	0.661	0.965	0.339	
74	0.293	0.483	0.646	0.742	0.3537	
73	0.792	0.864	0.896	0.932	0.1039	
72	0.437	0.372	0.719	0.686		0.3142
71	0.097	0.799	0.548	0.899	0.4516	
70	0.086	0.238	0.543	0.619	0.4571	
69	0.878	0.791	0.939	0.895		0.1046
68	0.84	0.4	0.92	0.7		0.2999
67	0.301	0.02	0.65	0.51		0.4899
66	0.419	0.685	0.71	0.842	0.2904	
65	0.302	0.766	0.651	0.883	0.3488	
64	0.315	0.071	0.657	0.535		0.4646
63	0.105	0.706	0.553	0.853	0.4473	
62	0.275	0.065	0.637	0.533		0.4674
61	0.458	0.344	0.729	0.672		0.3279
60	0.966	0.602	0.983	0.801		0.1992
59	0.808	0.931	0.904	0.966	0.096	
58	0.296	0.384	0.648	0.692	0.3518	
57	0.796	0.765	0.898	0.882		0.1177
56	0.176	0.866	0.588	0.933	0.4121	
55	0.019	0.178	0.51	0.589	0.4904	
54	0.323	0.737	0.661	0.868	0.3387	
53	0.374	0.808	0.687	0.904	0.3128	
52	0.336	0.418	0.668	0.709	0.332	
51	0.099	0.149	0.549	0.574	0.4507	
50	0.141	0.275	0.571	0.637	0.4294	
49	0.772	0.863	0.886	0.931	0.1139	
48	0.785	0.167	0.892	0.584		0.4163
47	0.575	0.802	0.788	0.901	0.2124	
46	0.511	0.564	0.756	0.782	0.2443	
45	0.462	0.245	0.731	0.622		0.3776
44	0.21	0.126	0.605	0.563		0.4372
43	0.052	0.455	0.526	0.728	0.474	
42	0.039	0.912	0.519	0.956	0.4805	
41	0.515	0.363	0.758	0.682		0.3183
40	0.662	0.867	0.831	0.933	0.1691	
39	0.82	0.298	0.91	0.649		0.351
38	0.123	0.857	0.562	0.928	0.4384	
37	0.674	0.932	0.837	0.966	0.1632	
36	0.134	0.546	0.567	0.773	0.4331	
35	0.342	0.673	0.671	0.836	0.3288	
34	0.165	0.114	0.583	0.557		0.4431

33	0.783	0.789	0.892	0.895	0.1084	
32	0.56	0.504	0.78	0.752		0.2479
31	0.327	0.209	0.663	0.605		0.3954
30	0.997	0.565	0.999	0.782		0.2177
29	0.181	0.844	0.59	0.922	0.4096	
28	0.932	0.716	0.966	0.858		0.1422
27	0.538	0.456	0.769	0.728		0.2719
26	0.525	0.913	0.762	0.957	0.2375	
25	0.025	0.294	0.512	0.647	0.4877	
24	0.404	0.395	0.702	0.698		0.3025
23	0.064	0.822	0.532	0.911	0.4682	
22	0.318	0.668	0.659	0.834	0.341	
21	0.111	0.221	0.555	0.61	0.4446	
20	0.807	0.424	0.903	0.712		0.2882
19	0.281	0.957	0.641	0.978	0.3595	
18	0.402	0.612	0.701	0.806	0.2989	
17	0.269	0.79	0.635	0.895	0.3654	
16	0.547	0.501	0.774	0.75		0.2497
15	0.492	0.204	0.746	0.602		0.3982
14	0.241	0.089	0.621	0.544		0.4557
13	0.425	0.368	0.712	0.684		0.3162
12	0.418	0.716	0.709	0.858	0.2908	
11	0.26	0.046	0.63	0.523		0.4769
10	0.277	0.32	0.638	0.66	0.3617	
9	0.776	0.701	0.888	0.85		0.1495
8	0.422	0.209	0.711	0.604		0.3957
7	0.314	0.234	0.657	0.617		0.3831
6	0.04	0.258	0.52	0.629	0.48	
5	0.62	0.151	0.81	0.576		0.4244
4	0.316	0.354	0.658	0.677	0.3419	
3	0.791	0.887	0.895	0.944	0.1047	
2	0.309	0.649	0.654	0.824	0.3456	
1	0.008	0.845	0.504	0.922	0.496	
			Win Prof 1	Win Prof 2	Σ	Σ
			0.386	0.268	19.298	13.401

2. Three-bidder uniform cost distribution

Scenario 1

high	1	low	0	3	Bidders	Equilibrium			
Cost1	Cost2	Cost3	Bid1	Bid2	Bid3	Payoff1	Payoff2	Payoff3	
									1
0.844	0.216	0.36	0.896	0.477	0.573		0.2613		1
0.961	0.953	0.575	0.974	0.968	0.717			0.1417	1
0.262	0.839	0.486	0.508	0.893	0.657	0.246			1
0.213	0.074	0.708	0.475	0.383	0.805		0.3086		1
0.583	0.044	0.997	0.722	0.363	0.998		0.3187		1
0.769	0.365	0.084	0.846	0.576	0.389			0.3055	1
0.381	0.347	0.617	0.588	0.564	0.745		0.2178		1
0.503	0.556	0.453	0.669	0.704	0.636			0.1822	1
0.219	0.447	0.101	0.48	0.631	0.4			0.2998	1
0.694	0.037	0.972	0.796	0.358	0.981		0.3211		1
0.954	0.172	0.769	0.969	0.448	0.846		0.2762		1
0.856	0.698	0.676	0.904	0.799	0.784			0.1079	1
0.059	0.665	0.48	0.373	0.777	0.654	0.314			1
0.592	0.874	0.118	0.728	0.916	0.412			0.294	1
0.44	0.476	0.815	0.627	0.65	0.877	0.187			1
0.779	0.383	0.508	0.853	0.589	0.672		0.2056		1

0.47	0.426	0.469	0.647	0.617	0.646		0.1913		1
0.677	0.623	0.112	0.785	0.749	0.408			0.2958	1
0.927	0.814	0.652	0.951	0.876	0.768			0.1161	1
0.889	0.979	0.188	0.926	0.986	0.459			0.2707	1
0.864	0.318	0.821	0.909	0.545	0.881		0.2274		1
0.36	0.784	0.249	0.573	0.856	0.5			0.2502	1
0.171	0.573	0.155	0.447	0.715	0.437			0.2817	1
0.101	0.932	0.82	0.401	0.954	0.88	0.3			1
0.642	0.876	0.724	0.761	0.917	0.816	0.119			1
0.066	0.771	0.224	0.377	0.847	0.483	0.311			1
0.353	0.74	0.764	0.569	0.827	0.842	0.216			1
0.454	0.074	0.043	0.636	0.382	0.362			0.3191	1
0.216	0.688	0.932	0.478	0.792	0.955	0.261			1
0.444	0.76	0.133	0.629	0.84	0.422			0.2891	1
0.964	0.295	0.181	0.976	0.53	0.454			0.273	1
0.377	0.759	0.874	0.585	0.839	0.916	0.208			1
0.627	0.449	0.921	0.751	0.633	0.947		0.1837		1
0.517	0.544	0.022	0.678	0.696	0.348			0.3261	1
0.044	0.072	0.047	0.363	0.382	0.365	0.319			1
0.487	0.355	0.555	0.658	0.57	0.703		0.215		1
0.224	0.121	0.759	0.483	0.414	0.839		0.2928		1
0.466	0.111	0.33	0.644	0.408	0.553		0.2962		1
0.3	0.796	0.656	0.533	0.864	0.77	0.233			1
0.902	0.088	0.672	0.935	0.392	0.781		0.304		1
0.674	0.648	0.804	0.783	0.765	0.87		0.1174		1
0.906	0.692	0.133	0.938	0.795	0.422			0.2889	1
0.657	0.875	0.216	0.772	0.916	0.477			0.2613	1
0.317	0.855	0.581	0.545	0.904	0.721	0.228			1
0.904	0.523	0.727	0.936	0.682	0.818		0.1589		1
0.675	0.339	0.585	0.783	0.559	0.723		0.2203		1
0.156	0.144	0.367	0.437	0.429	0.578		0.2854		1
0.377	0.757	0.89	0.584	0.838	0.926	0.208			1
0.174	0.129	0.916	0.449	0.42	0.944		0.2902		1
0.623	0.872	0.102	0.749	0.914	0.401			0.2993	1
0.915	0.745	0.985	0.943	0.83	0.99		0.0849		1
0.125	0.653	0.591	0.416	0.769	0.728	0.292			1
0.389	0.97	0.493	0.593	0.98	0.662	0.204			1
0.487	0.575	0.426	0.658	0.717	0.617			0.1915	1
0.287	0.21	0.098	0.525	0.473	0.398			0.3008	1
0.272	0.995	0.465	0.514	0.997	0.644	0.243			1
0.64	0.722	0.973	0.76	0.814	0.982	0.12			1
0.96	0.996	0.891	0.973	0.997	0.928			0.0362	1
0.049	0.9	0.348	0.366	0.934	0.566	0.317			1
0.403	0.467	0.703	0.602	0.645	0.802	0.199			1

0.79	0.048	0.919	0.86	0.366	0.946		0.3172		1
0.003	0.961	0.966	0.335	0.974	0.977	0.332			1
0.365	0.23	0.78	0.577	0.487	0.853		0.2567		1
0.104	0.482	0.548	0.403	0.655	0.699	0.299			1
0.143	0.19	0.142	0.428	0.46	0.428			0.286	1
0.453	0.521	0.803	0.635	0.681	0.868	0.182			1
0.212	0.407	0.346	0.475	0.604	0.564	0.263			1
0.196	0.19	0.729	0.464	0.46	0.819		0.2701		1
0.457	0.094	0.147	0.638	0.396	0.431		0.3021		1
0.785	0.314	0.307	0.857	0.543	0.538			0.2309	1
0.348	0.377	0.55	0.565	0.585	0.7	0.217			1
0.531	0.969	0.29	0.687	0.979	0.527			0.2366	1
0.911	0.626	0.112	0.941	0.751	0.408			0.2959	1
0.461	0.516	0.259	0.641	0.677	0.506			0.2471	1
0.483	0.821	0.916	0.656	0.881	0.944	0.172			1
0.916	0.883	0.593	0.944	0.922	0.729			0.1356	1
0.608	0.67	0.829	0.739	0.78	0.886	0.131			1
0.504	0.481	0.333	0.67	0.654	0.555			0.2223	1
0.799	0.15	0.348	0.866	0.434	0.565		0.2832		1
0.363	0.456	0.373	0.575	0.638	0.582	0.212			1
0.961	0.812	0.052	0.974	0.874	0.368			0.316	1
0.61	0.105	0.884	0.74	0.403	0.923		0.2983		1
0.177	0.14	0.256	0.452	0.427	0.504		0.2867		1
0.319	0.948	0.076	0.546	0.965	0.384			0.3079	1
0.228	0.433	0.169	0.485	0.622	0.446			0.277	1
0.207	0.743	0.931	0.472	0.829	0.954	0.264			1
0.126	0.673	0.774	0.417	0.782	0.849	0.291			1
0.399	0.725	0.942	0.599	0.816	0.961	0.2			1
0.666	0.061	0.707	0.777	0.374	0.804		0.313		1
0.68	0.164	0.396	0.787	0.443	0.598		0.2785		1
0.34	0.646	0.254	0.56	0.764	0.502			0.2488	1
0.722	0.013	0.559	0.814	0.342	0.706		0.329		1
0.956	0.543	0.452	0.971	0.695	0.635			0.1825	1
0.802	0.659	0.585	0.868	0.772	0.723			0.1384	1
0.921	0.382	0.857	0.948	0.588	0.904		0.2059		1
0.169	0.587	0.339	0.446	0.724	0.559	0.277			1
0.84	0.498	0.018	0.894	0.665	0.345			0.3273	1
0.173	0.69	0.308	0.449	0.793	0.538	0.276			1
0.831	0.185	0.108	0.887	0.456	0.405			0.2973	1
0.921	0.297	0.556	0.947	0.531	0.704		0.2343		1
			B 1	B 2	B 3	T pr.1	T pr 2	T pr 3	
Average Prof			0.076	0.082	0.089	7.64	8.152	8.8805	100
Winning Prof			0.239	0.255	0.247	32	32	36	

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